Name____

Name____

Elementary Fluid Mechanics CEE 357-02
Fall 2019- December 06
Exam 3

Circle the correct answer or fill in the blank

1. (**3pt**) Calculation of the wetted Perimeter (P) requires summation of______________ inside a channel for which water flows through a fixed cross section:

(a) all of the sides, bottom, and free surface
(b) the sides and the bottom, but not the free surface
(c) only the portion of the sides in contact with water, and the bottom

2. (**3pt**) Surface wave speed can be used to calculate the
(a) Mach number
(b) Froude number
(c) Prandtl number

3. (**3pt**) The classification scheme that allows for the distinction between gradually varying flow and rapidly varying flow requires that we assume____________ for gradually varying flow and ____________ for rapidly varying flow (phrase).

One-dimensional approximation, multidimensional flow

Solve and show your work.

4. (**18 pts**) Water flows down a rectangular channel that is 4 ft wide and 2 ft deep. The flow rate is 28,000 gal/min. Estimate the Froude number of the flow. Round the answer to two decimal places.

Using BG units, convert 28,000 gallons/min to ft^3/s.

$$28{,}000\ \frac{gal}{min} \times \frac{1\ ft^3/s}{448.8\ gal/min} = 62.38\ ft^3/s$$

The velocity and the wave speed are:

$$V = \frac{62.38\ ft^3/s}{(4\ ft)(2\ ft)} = 7.80\ ft/s \qquad (6\ pts)$$

$$c_0 = \sqrt{gy} = \sqrt{(32.2)\times(2ft)} = 8.02\ \frac{ft}{s} \qquad (6\ pts)$$

$$Froude\ number = Fr = \frac{V}{c_0} = \frac{7.80\ ft/s}{8.02\ \frac{ft}{s}} \approx 0.97 \qquad (6pts)$$

Name____

Circle the correct answer

5. (**3 pt**) An elementary wave traveling on the surface of a fluid within a subcritical regime would _____________relative to the flow of the fluid.

(a) flow downstream

(b) remain stationary

(c) flow upstream

6. (**3 pt**) The best of all possible channel cross-sections (maximizing R_h, minimizing P) is _______.

(a) a square with the relation b = 2y

(b) a semi-circle

(c) a trapezoid

7. (**3 pt**) Supercritical flow requires a height (y) above the channel bottom that is _______.

(a) greater than the critical depth (y_c)

(b) less than the critical depth (y_c)

(c) equal to the critical depth (y_c)

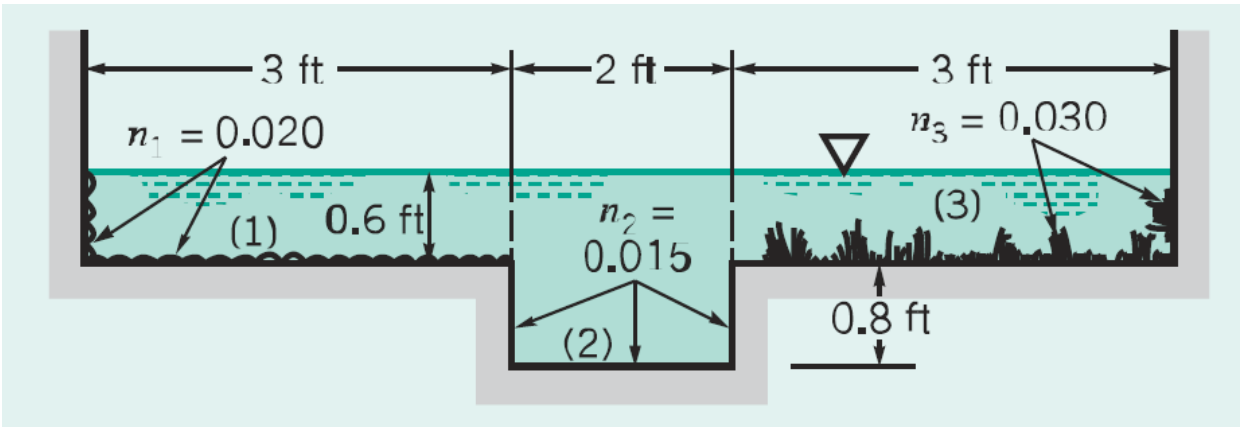
Name____

Solve and show your work.

8. Uniform flow in a Channel. (**28 pts**)

Water flows along the drainage canal having properties shown in the Figure. The bottom slope is S_o = 1ft/500 ft = 0.002, with variable Manning roughness coefficients (n_1= 0.20, n_2 = 0.015, n_3= 0.030) for the three sub-sections of flow in the channel.

Estimate the flow rate in ft^3/s.



Divide the cross section into three subsections as indicated in the Figure and write:

$$Q = Q_1 + Q_2 + Q_3, \text{ where for each section :}$$

$$Q_i = \frac{1.49}{n_i} A_i R_{hi}^{2/3} S_o^{1/2}$$

The appropriate values of $A_i, P_i, R_{hi}, and\ n_i$ are:

$$A_1 = (3\,ft)(0.6\,ft) = 1.8\,ft^2\,;\ \ A_2 = (2\,ft)(0.8 + 0.6) = 2.8\,ft^2\,;\ \ \ A_3 = (3\,ft)(0.6\,ft) = 1.8\,ft^2$$

(6pts)

$$P_1 = (3\,ft) + (0.6\,ft) = 3.6\,ft;\ P_2 = (2\,ft) + 2(0.8) = 3.6\,ft;\ P_3 = (3\,ft) + (0.6\,ft) = 3.6\,ft$$

(6pts)

Note that the wetted perimeter P_2 is only the channel portion …

$$R_{h1} = \frac{1.8\,ft^2}{3.6\,ft} = 0.50\,ft;\ \ R_{h2} = \frac{2.8\,ft^2}{3.6\,ft} = 0.778\,ft;\ R_{h3} = \frac{1.8\,ft^2}{3.6\,ft} = 0.50\,ft$$

(6pts)

$$n_1 = 0.02\,;\ n_2 = 0.015;\ \ n_3 = 0.030$$

$$Q = Q_1 + Q_2 + Q_3$$

Name____

$$Q = 1.49\ (0.002)^{1/2}\ \times \left[\frac{(1.8ft^2)(0.5ft)^{2/3}}{0.020} + \frac{(2.8ft^2)(0.778ft)^{2/3}}{0.015} + \frac{(1.8ft^2)(0.5ft)^{2/3}}{0.030}\right]$$

$$(5\ pts)$$

$$Q_{total} =\ 1.49\ (0.0447) \times [\ 56.70 +\ 157.87 +\ 37.80]$$

$$Q_{total} =\ 1.49\ (0.0447) \times [\ 252.37] = 16.82\ \ ft^3/s \qquad (5\ pts)$$

Circle the correct answer

9. (**2 pt**) A supercritical flow regime will__________ when flowing over a bump:
(a) stay the same depth
(b) decrease in depth
(c) increase in depth

10. (2 **pt**) A subcritical flow at height (y_1) that encounters a sluice gate and smoothly accelerates below the gate will contract to a new height (y_2) that is about ___________________________ less than the gate height (H).
(a) 2/3rds
(b) 40 percent
(c) 60 percent.

11. (2 **pt**) An unstable, oscillating hydraulic jump has a range of Froude numbers between: ___________________________.

(a) Fr =2.5 to 4.5
(b) Fr = 1.7 to 2.5
(c) Fr > 9.0

Name____

Solve and show your work.

12. Flow Measurements controlled by Weirs **(30 Pts)**

A weir in a horizontal channel is 4 m high and 16 m wide. The water depth upstream is 4.6 m. Estimate the discharge if the weir is (a) sharp-crested and (b) round-nosed with an unfinished concrete broad crest 1.2 m long. Neglect $V_1^2/(2g)$.

Given that $Y = 4$ and $H + Y \approx 4.6$, hence $H \approx 0.6$

Since $H \ll b$, we assume that the weir is "wide". For a sharp crest, Eq below applies:

$$C_d \approx 0.564 + 0.0846\,\frac{H}{Y} \quad for \quad \frac{H}{Y} \leq 2$$

$$C_d \approx 0.564 + 0.0846\,\frac{0.6\ m}{4\ m} = 0.577 \qquad (5pts)$$

(a) $$Q = C_d b\,\sqrt{g}H^{3/2} = (0.577)(16m)\sqrt{(9.81\ m/s^2)}(0.6m)^{3/2} \approx 13.43\ m^3/s \qquad (10pts)$$

For a round-nosed broad-crested weir, the following equations below apply. For an unfinished concrete surface, read $\varepsilon = 2.4\ mm$ from the table in the book (Table 10.1).

The displacement thickness is:

$$\frac{\delta^*}{L} \approx 0.001 + 0.2\,\sqrt{\varepsilon/L} = 0.001 + 0.2\,\sqrt{(0.0024m/1.2m)} \approx 0.00994 \qquad (2pts)$$

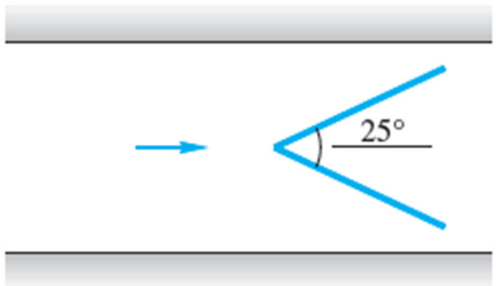
$$C_d \approx 0.544\left(1 - \frac{\delta^*/L}{H/L}\right)^{3/2} \approx 0.544\left(1 - \frac{0.00994}{0.6m/1.2m}\right)^{3/2} \approx 0.528 \qquad (3pts)$$

$$Q = C_d b\,\sqrt{g}H^{3/2} = (0.528)(16m)\sqrt{(9.81\ m/s^2)}(0.6m)^{3/2} \approx 12.29\ m^3/s \qquad (10pts)$$

Name____

---------------BONUS----------------------

Bonus: A pencil point piercing the surface of a rectangular channel flow creates a 25° half-angle wedge like wave, as shown in the image below. The channel surface is painted steel and the depth is 65 cm.



(2pts) Determine the Froude number. Round the final answer to two decimal places

Using the relation for the angle of the waves:

$$\mu = \sin^{-1}\left(\frac{1}{Fr}\right) \ or \ Fr = \frac{1}{\sin\mu} \ ; \ Fr = \csc\mu = \csc(25°) = 2.37$$

(2pts) Determine the critical depth. Round the final answer to two decimal places.

$$V = Fr \times V_c = 2.37\sqrt{(9.81)(0.65)} = 5.98 \ ft/s$$

$$Flow\ rate \quad q = Vy = 5.98\ (0.65) = 3.88\frac{m^2}{s}$$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{(3.88)^2}{9.81\ m/s^2}\right)^{1/3} \approx 1.15\ m$$

Bonus (2 points): A cast iron duct (n =0.013) of diameter 2.9 m is flowing half-full at 33 m^3/s. Determine the slope of this duct if the flow is uniform. Round answer to four decimal places.

$$Q = \frac{1.0}{n}AR_h^{2/3}S_o^{1/2}$$

$$P = \pi R = \pi(1.45m)$$

$$R_h = \frac{A}{P} = \frac{\pi R^2/2}{\pi R} = 0.725$$

$$S_o^{1/2} = \frac{Qn}{AR_h^{2/3}} = \frac{(33\ m^3/s)(0.013)}{(\frac{\pi(1.45)^2}{2})(0.725^{2/3})} = 0.16096$$

$$S_o = 0.16096^2 = 0.02591$$

Bonus (2 points): Find the flow rate (in cubic ft per second) of a half-full glass circular pipe with a slope of 2° and diameter 4.5 ft.

$$Q = \frac{1.49}{n}AR_h^{2/3}S_o^{1/2}$$

$$P = \pi R = \pi(2.25m)$$

$$R_h = \frac{A}{P} = \frac{\pi R^2/2}{\pi R} = \frac{2.25}{2} = 1.125$$

$$Q = \frac{1.49}{0.010}(\pi(2.25)^2/2)(1.125)^{2/3}(\tan 2°)^{1/2} = 239.51\ ft^3/s$$