Homework 1 Solutions

1.1

$$m = cV = 8500\frac{\text{g}}{\text{m}^3}\cdot 2.5\ \text{L}\cdot\frac{1\ \text{m}^3}{10^3\ \text{L}} = 21.25\ \text{g}$$

1.2

(*a*) $Q = 100{,}000\ \text{capita}(650\ \text{L}/\text{capita}/\text{d})\frac{1\ \text{m}^3}{10^3\ \text{L}}\frac{\text{d}}{86400\ \text{s}} = 0.7523\frac{\text{m}^3}{\text{s}}$

$$W = 100{,}000\ \text{capita}(135\ \text{grams}/\text{capita}/\text{d})\cdot\frac{365\ \text{d}}{\text{yr}}\cdot\frac{\text{kg}}{10^3\ \text{g}}\cdot\frac{1\ \text{mta}}{10^3\ \text{kg}} = 4927.5\ \text{mta}$$

(*b*) $c = \frac{W}{Q} = \frac{135\ \text{g}/\text{capita}/\text{d}}{650\ \text{L}/\text{capita}/\text{d}}\cdot\frac{10^3\ \text{L}}{1\ \text{m}^3} = 207.7\frac{\text{mg}}{\text{L}}$

1.4

(*a*) $a = \frac{W}{c} = \frac{6{,}950\ \text{mta}}{8\ \text{mg}/\text{m}^3}\cdot\frac{10^9\ \text{mg}}{\text{tonne}}\cdot\frac{\text{km}^3}{10^9\ \text{m}^3} = 868.75\frac{\text{km}^3}{\text{yr}}$

(*b*) $W = ac = 868.75\frac{\text{mta}}{\mu\text{g}/\text{L}}\cdot 5\ \mu\text{g}/\text{L} = 4343.75\ \text{mta}$

(c) $\%\ \text{reduction} = \frac{6{,}950 - 4{,}343.75}{6{,}950}\cdot 100\% = 37.5\%$

1.7 For the present case, the following balance must hold

$$Q_A(0.5 \text{ g / L}) + Q_B(0.05 \text{ g / L}) = 4 \text{ cms}(0.1 \text{ g / L})$$

Because this equation has two unknowns, another equation is required. This additional formulation arises from the fact that

$$Q_A + Q_B = 4 \text{ cms}$$

This equation can be solved for Q_B and the result substituted into the first equation to give

$$Q_A(0.5 \text{ g/L}) + (4 - Q_A)(0.05 \text{ g/L}) = 4 \text{ cms}(0.1 \text{ g/L})$$

which can be solved for

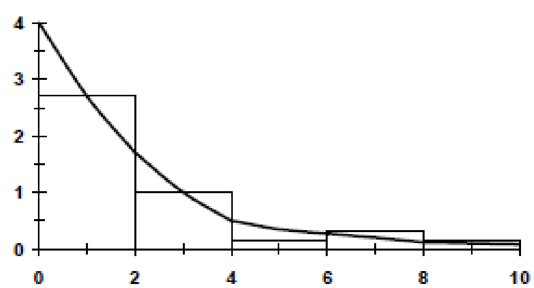
$$Q_A = \frac{0.2}{0.45} = 0.444 \text{ cms}$$

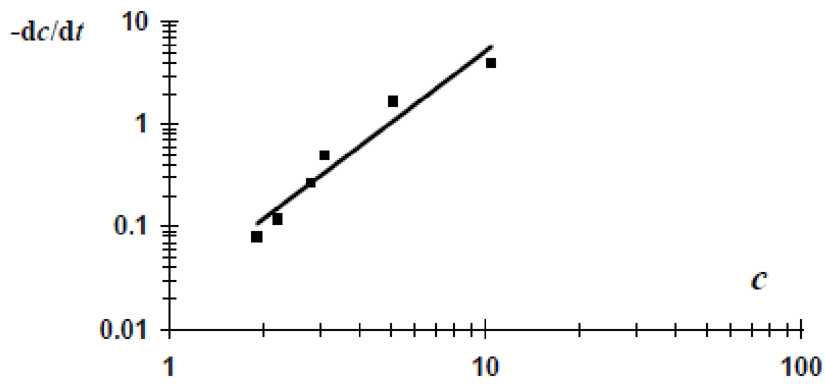
and

$$Q_B = 4 - 0.444 = 3.556 \text{ cms}$$

2.1 Applying equal-area differentiation

t (d)	c (mg/L)	$-\frac{\Delta c}{\Delta t}$ (mg L^{-1} d^{-1})	$-\frac{dc}{dt}$
0	10.5		4
		2.7	
2	5.1		1.7
		1	
4	3.1		0.5
		0.15	
6	2.8		0.27
		0.3	
8	2.2		0.12
		0.15	
10	1.9		0.8





Linear regression yields a least-squares fit of

$$\log\left(-\frac{dc}{dt}\right) = -1.6213 + 2.333\log c$$

Therefore according to the differential approach, the parameters are estimated as n = 2.333 and k = 0.0239.

On the basis of this result, we can try the integral method with $n = 2$. A fit of the plot of $1/c$ versus t yields a best-fit line of

$$\frac{1}{c} = 0.1135 + 0.04236t$$

Therefore, this fit yields $k = 0.04236$.

Finally, a nonlinear regression can be used to directly fit the model (see Lec. 8 problems for additional details on how this is done). The result is

$$\frac{dc}{dt} = -0.0326c^{2.23}$$

The results can be summarized as

	integral	integral/least-squares	differential
order	2	2.23	2.333
rate	0.04236	0.0326	0.0239

Thus, the integral/least-squares method falls between the integral and differential results.

2.5

$$\theta = Q_{10}^{0.1} = 1.9^{0.1} = 1.066$$

$$k_{30} = 1.6(1.066)^{30-20} = 3.04 \text{ d}^{-1}$$

3.4 **(*a*)** $Q = \dfrac{V}{\tau_w} = \dfrac{1000000}{0.75} = 1{,}333{,}333 \dfrac{\text{m}^3}{\text{yr}}$ $\quad c_{in} = \dfrac{W}{Q} = \dfrac{1\times 10^7}{1.33\times 10^6} = 7.5 \dfrac{\text{mg}}{\text{m}^3}$

(*b*) $\beta = \dfrac{c}{c_{\text{in}}} = \dfrac{0.8}{7.5} = 0.10667$

(*c*) $J_v = \dfrac{W - Qc}{A_s} = \dfrac{1\times 10^7 - 1.333\times 10^6(0.8)}{1\times 10^5} = 89.33 \dfrac{\text{mg}}{\text{m}^2\,\text{yr}}$

(*d*) $v_v = \dfrac{J_v}{c} = \dfrac{89.33}{0.75} = 111.667 \dfrac{\text{m}}{\text{yr}} = 0.305 \dfrac{\text{m}}{\text{d}}$

4.5 The lake's eigenvalue can be computed as

$$\lambda = \frac{Q}{V} + \frac{v}{H} = \frac{5\times 10^5}{4\times 10^7} + \frac{8}{8} = \frac{1.0125}{\text{yr}}$$

The solution for the step loading is

$$c = \frac{W}{\lambda V}\left(1 - e^{-\lambda t}\right) = \frac{5\times 10^8}{1.0125(4\times 10^7)}\left(1 - e^{-1.0125(t-1994)}\right) = 12.346\left(1 - e^{-1.0125(t-1994)}\right)$$

This equation can be used to compute the values for the lake over time. These values, along with the base concentration of 5 μg/L are shown in the following plot