P6.109 In Fig. P6.109 there are 125 ft of 2-in pipe, 75 ft of 6-in pipe, and 150 ft of 3-in pipe, all cast iron. There are two 90° elbows and an open globe valve, all flanged. If the exit elevation is zero, what horsepower is extracted by the turbine when the flow rate is 0.16 ft^3/s of water at 20°C?

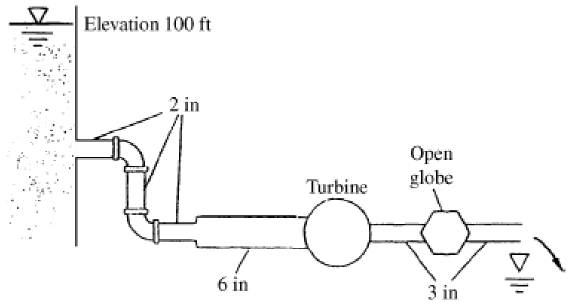


Fig. P6.109

Solution: For water at 20°C, take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09E{-}5$ slug/ft·s. For cast iron, $\varepsilon \approx 0.00085$ ft. The 2″, 6″, and 3″ pipes have, respectively,

$$\text{(a)}\quad L/d = 750,\ \varepsilon/d = 0.0051; \quad \text{(b)}\quad L/d = 150,\ \varepsilon/d = 0.0017;$$

$$\text{(c)}\quad L/d = 600,\ \varepsilon/d = 0.0034$$

The flow rate is known, so each velocity, Reynolds number, and *f* can be calculated:

$$V_a = \frac{0.16}{\pi(2/12)^2/4} = 7.33\ \frac{ft}{s}; \quad Re_a = \frac{1.94(7.33)(2/12)}{2.09E{-}5} = 113500, \quad f_a \approx 0.0314$$

$$\text{Also, } V_b = 0.82 \text{ ft/s},\ Re_b = 37800,\ f_c \approx 0.0266;\ V_c = 3.26,\ Re_c = 75600,\ f_c \approx 0.0287$$

Finally, the minor loss coefficients may be tabulated:

sharp 2″ entrance: K = 0.5; two 2″ 90° elbows: K = 2(0.39)

2″ sudden expansion: K ≈ 0.79; 3″ open globe valve: K ≈ 8.5

The turbine head equals the elevation difference minus losses and the exit velocity head:

$$h_t = \Delta z - \sum h_f - \sum h_m - V_c^2/(2g)$$

$$= 100 - \frac{(7.33)^2}{2(32.2)}[0.0314(750) + 0.5 + 2(0.39) + 0.79]$$

$$- \frac{(0.82)^2}{2(32.2)}(0.0266)(150) - \frac{(3.26)^2}{2(32.2)}[0.0287(600) + 8.5 + 1] \approx \mathbf{74.2\ ft}$$

The resulting turbine power = $\rho g Q h_t = (62.4)(0.16)(74.2) \div 550 \approx$ **1.35 hp**. *Ans.*

P6.112 If the two pipes in Fig. P6.111 are instead laid in **series** with the same total pressure drop of 3 psi, what will the flow rate be? The fluid is SAE 10 oil at 20°C.

Solution: For SAE 10 oil at 20°C, take ρ = 1.69 slug/ft^3 and μ = 0.00217 slug/ft·s. Again guess laminar flow. Now, instead of Δp being the same, Qa = Qb = Q:

$$\Delta p_a + \Delta p_b = 432 \text{ psf} = \frac{128\mu L_a Q}{\pi d_a^4} + \frac{128\mu L_b Q}{\pi d_b^4} = \frac{128(0.00217)}{\pi} Q\left[\frac{250}{(3/12)^4} + \frac{200}{(2/12)^4}\right]$$

Solve for $\mathbf{Q \approx 0.0151\ ft^3/s}$ *Ans.* Check $\text{Re}_a \approx 60$ (OK) and $\text{Re}_b \approx 90$ (OK)

In series, the flow rate is six times less than when the pipes are in parallel.

P6.156 Ethanol at 20°C flows down through a modern venturi nozzle as in Fig. P6.156. If the mercury manometer reading is 4 in, as shown, estimate the flow rate, in gal/min.

Solution: For ethanol at 20°C, take ρ = 1.53 slug/ft^3 and μ = 2.51E–5 slug/ft·s. Given β = 0.5, the discharge coefficient is

$$C_d = 0.9858 - 0.196(0.5)^{4.5} \approx 0.9771$$

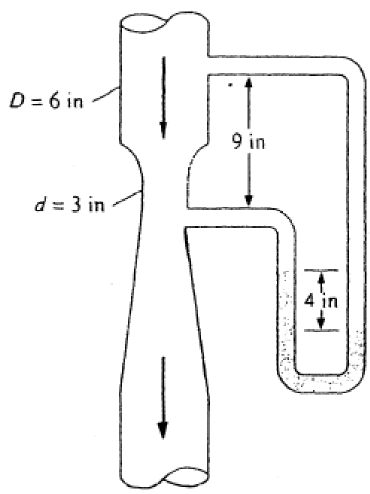


Fig. P6.156

The 9-inch displacement of manometer taps does not affect the pressure drop reading, because both legs are filled with ethanol. Therefore we proceed directly to Δp and Q:

$$\Delta p_{nozzle} = (\rho_{merc} - \rho_{eth})gh = (26.3 - 1.53)(32.2)(4/12) \approx 266 \text{ lbf/ft}^2$$

Hence $$Q = C_d A_t\left[\frac{2\,\Delta p}{\rho(1-\beta^4)}\right]^{1/2} = 0.9771\left(\frac{\pi}{4}\right)\left(\frac{3}{12}\right)^2\sqrt{\frac{2(266)}{1.53(1-0.5^4)}} \approx \mathbf{0.924}\ \frac{ft^3}{s}$$ *Ans.*

P6.159 A modern venturi nozzle is tested in a laboratory flow with water at 20°C. The pipe diameter is 5.5 cm, and the venturi throat diameter is 3.5 cm. The flow rate is measured by a weigh tank and the pressure drop by a water-mercury manometer. The mass flow rate and manometer readings are as follows:

$\dot{m}$, kg/s:	0.95	1.98	2.99	5.06	8.15
h, mm:	3.7	15.9	36.2	102.4	264.4

Use these data to plot a calibration curve of venturi discharge coefficient versus Reynolds number. Compare with the accepted correlation, Eq. (6.114).

Solution: For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. The given data of mass flow and manometer height can readily be converted to discharge coefficient and Reynolds number:

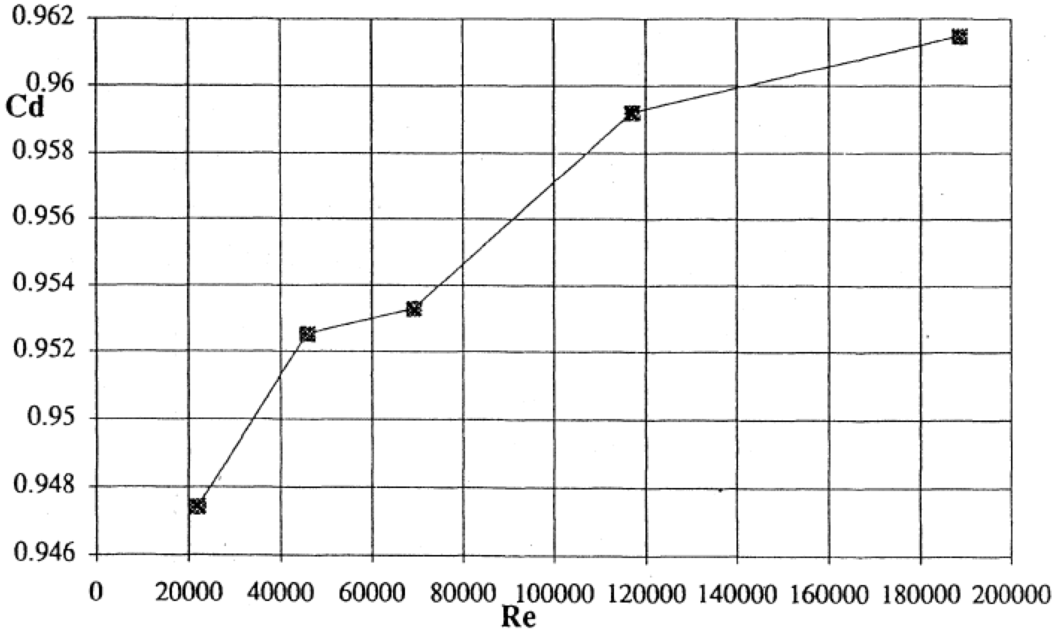
$$Q = \frac{\dot{m}}{998} = C_d\left(\frac{\pi}{4}\right)(0.035)^2\sqrt{\frac{2(13.56-1)\rho_w(9.81)h}{\rho_w[1-(3.5/5.5)^4]}}, \quad \text{or:} \quad C_d \approx \frac{\dot{m}\ (\text{kg/s})}{16.485\sqrt{h_{meters}}}$$

$$Re_D = \frac{4\,\dot{m}}{\pi\mu D} = \frac{4\,\dot{m}}{\pi(0.001)(0.055)} \approx 23150\,\dot{m}\ (\text{kg/s})$$

The data can then be converted and tabulated as follows:

h, m:	0.037	0.0159	0.0362	0.1024	0.2644
Cd:	0.947	0.953	0.953	0.959	0.962
ReD:	22000	46000	69000	117000	189000

These data are plotted in the graph below, similar to Fig. 6.42 of the text:



They closely resemble the "classical Herschel venturi," but this data is actually for a *modern* venturi, for which we only know the value of C_d for $1.5E5 < Re_D \le 2E5$:

$$\text{Eq. (6.116)} \quad C_d \approx 0.9858 - 0.196\left(\frac{3.5}{5.5}\right)^{4.5} \approx \mathbf{0.960}$$

The two data points near this Reynolds number range are quite close to 0.960 ± 0.002.