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Elementary Fluid Mechanics CEE 357-02
Fall 2019- November 18
Exam 2

Circle the correct answer or fill in the blank

1. (2pt) The region where an inviscid core velocity converges, viscous boundary layers grow, and the entrance pressure drops nonlinearly with x (for either laminar and turbulent flow) is referred to as:

- (a) the entrance region
- (b) pipe entrance
- (c) fully developed flow region

2. (2pt) Turbulent shear is dominant near

- (a) the intermediate region called the overlap layer
- (b) the region referred to as the wall layer
- (c) the region referred to as the outer layer

3. (2pt) When comparing the laminar theory for noncircular ducts to the flow solution for circular ducts, the friction factor would be lower by a proportion of _____ (numerical value) if the hydraulic diameter was simply inserted into the Poiseuille flow friction factor equation (for circular pipe).

33%

Solve and show your work.

4. (24 pts) Seawater (30%) is flowing through a 25-cm pipe, 250 m long, with a head loss of 46 m at 20°C. The concrete pipe used to transmit the seawater is rough. Solve for the (a) final expected friction factor, (b) average velocity (m/s), and (c) flow rate (m³/s). (d) Plot the friction factor value on the Moody chart. Correct values will be accepted within 2% margin.

Concrete pipe (smoothed) roughness value (ϵ) = 0.04 mm

Relative roughness ratio ($\frac{\epsilon}{d}$) = $\frac{2}{250} = 0.008$

$$f = h_f \frac{d}{L} \frac{2g}{V^2} = (16 \text{ m}) \left(\frac{0.40 \text{ m}}{200 \text{ m}} \right) \left[\frac{2 \left(9.81 \frac{\text{m}}{\text{s}^2} \right)}{V^2} \right] \quad \text{or} \quad fV^2 = 0.9025 \text{ (SI units)}$$

Guess a f value to compute a value for velocity (V) from above:

$$V = \sqrt{0.9025/f}$$

With a guess of 0.014 (similar to example and other homework problem guesses):

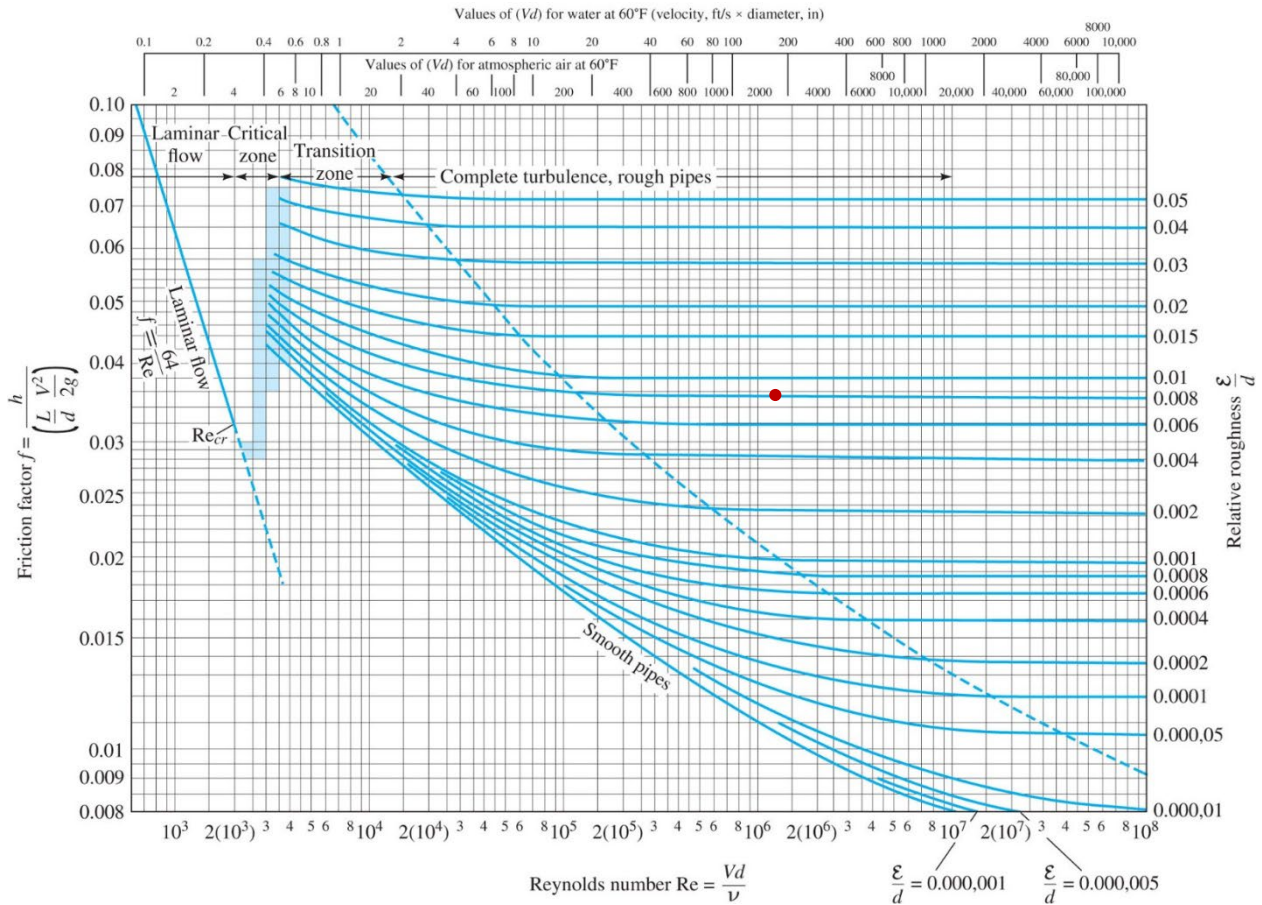
$$V = \sqrt{0.9025/0.014} = 8.0291 \text{ m/s}$$

We calculate a Reynolds number (Re_d) using this V

$$Re_d = \frac{Vd}{\nu} = \frac{\rho_{\text{Saltwater (30\%)}} Vd}{\mu_{\text{Saltwater (30\%)}}} = \frac{\left(8.029 \frac{\text{m}}{\text{s}} \right) (0.25 \text{ m})}{1.044 \text{ E} - 6 \text{ m}^2/\text{s}} = \frac{1025 \text{ kg/m}^3 \left(8.029 \frac{\text{m}}{\text{s}} \right) (0.25 \text{ m})}{1.07 \text{ E} - 3 \text{ kg}/(\text{m} \cdot \text{s})}$$

$Re_d = 1,922,845$

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Equation 6.48

$$\frac{1}{f^{1/2}} = -2.0 \log_{10} \left(\frac{\epsilon/d}{3.7} + \frac{2.51}{Re_d f^{1/2}} \right) \quad \text{or} \quad f = \left[1 / \left(-2 \log_{10} \left(\frac{\epsilon/d}{3.7} + \frac{2.51}{Re_d f^{1/2}} \right) \right) \right]^2$$

(a) $f = 0.035255 \pm 0.000705$ (6pts)

(b) With a new f value of 0.01261 velocity can be updated:

$$V = \sqrt{0.9025 / 0.0353} = 5.06 \frac{m}{s} \pm 0.101 \quad (6pts)$$

(c) Average flow rate (Q) m^3/s :

$$Q = V \left(\frac{\pi}{4} \right) d^2 = 7.056 \frac{m}{s} \times \left(\frac{\pi}{4} \right) (0.25m)^2 = 0.248 \frac{m^3}{s} \pm 0.00497 \quad (6pts)$$

(d) Friction factor plotted above at $f=0.0353$, $\left(\frac{\epsilon}{d}\right) = \frac{0.04}{400} = 0.0001$, $Re = 1,211,695$ (6pts)

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Circle the correct answer

5. (2 pt) The _____ model of the Reynolds stress terms, devised by L. Prandtl, provides for a mixing length that describes a parameter analogous to mean free path in molecular theory.

- (a) Darcy friction factor
- (b) Kármán constant
- (c) eddy viscosity

6. (2 pt) A measure of a pipe's thickness and its resistance to stress caused by internal fluid pressure is known as the _____.

- (a) Effective diameter mass flow
- (b) Logarithm law
- (c) Schedule No.

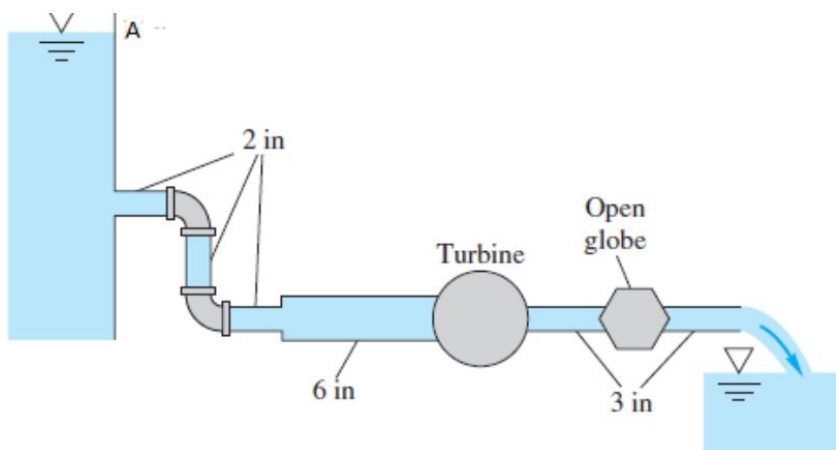
7. (2 pt) For laminar friction in noncircular ducts, our most accurate predictions come when we replace a parameter in the head loss equation with an approximation for the non-circular geometry which is referred to as _____.

- (a) the mixing length
- (b) the hydraulic diameter
- (c) the effective diameter

Solve and show your work.

8. Minor losses in a pipe. (28 pts)

In the figure shown below there are 325 ft of 2-in pipe, 175 ft of 6-in pipe, and 250 ft of 3-in pipe, all new cast iron. There are two 90° elbows and an open globe valve, **all flanged**. If the exit elevation is zero and the elevation at A is 150 ft, what horsepower is extracted by the turbine (in hp) when the flow rate is 0.20 ft³/s of water at 20 °C? Hint: To save time use Haaland's formula to calculate f .



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Properties of water at 20°C, $\rho = 1.94 \text{ slugs/ft}^3$, $\mu = 2.09E-5 \text{ slugs/(fts)}$. Roughness ratio and L/d calculations for the three sections of the pipe are: (2pts)

$$2'' = \frac{\varepsilon}{d} = \frac{0.00085 \text{ ft}}{\left(\frac{2}{12}\right)\text{ft}} = 0.0051 \quad \frac{L}{D} = \frac{325 \text{ ft}}{\left(\frac{2}{12}\right)\text{ft}} = 1950$$

$$6'' = \frac{\varepsilon}{d} = \frac{0.00085 \text{ ft}}{\left(\frac{6}{12}\right)\text{ft}} = 0.0017 \quad \frac{L}{D} = \frac{175 \text{ ft}}{\left(\frac{6}{12}\right)\text{ft}} = 350$$

$$3'' = \frac{\varepsilon}{d} = \frac{0.00085 \text{ ft}}{\left(\frac{3}{12}\right)\text{ft}} = 0.0034 \quad \frac{L}{D} = \frac{250 \text{ ft}}{\left(\frac{3}{12}\right)\text{ft}} = 1000$$

The velocities and the Reynolds numbers of the three pipes are determined using the following formula:

(2pts)

(2pts)

$$V_1 = \frac{0.2}{\pi(2/12)^2/4} = 9.167 \frac{\text{ft}}{\text{s}} ; \quad Re_1 = \frac{1.94(9.17)\left(\frac{2}{12}\right)}{2.09E-5} = 141823 ; \quad f_1(\text{Haaland}) = 0.0312$$

$$V_2 = \frac{0.2}{\pi(6/12)^2/4} = 1.019 \frac{\text{ft}}{\text{s}} ; \quad Re_1 = \frac{1.94(1.02)\left(\frac{6}{12}\right)}{2.09E-5} = 47274 ; \quad f_2(\text{Haaland}) = 0.0257$$

$$V_3 = \frac{0.2}{\pi(3/12)^2/4} = 4.074 \frac{\text{ft}}{\text{s}} ; \quad Re_1 = \frac{1.94(4.074)\left(\frac{3}{12}\right)}{2.09E-5} = 94548 ; \quad f_3(\text{Haaland}) = 0.0283$$

$K_1 = \text{entrance} = 0.5$; $K_2 = 2'' \text{ flanged } 90^\circ \text{ elbow} = 0.39$; $K_3 = 2'' \text{ flanged } 90^\circ \text{ elbow} = 0.39$;
 $K_4 = \text{expansion} \approx 0.79$; $K_5 = 3'' \text{ open globe valve} = 8.5 \text{ or } 6.0 \text{ (or between)}$; $K_6 = \text{exit} = 1.0$

(2pts)

The turbine head equals the elevation difference minus losses, such that (with $p_1=p_2=0$, and $V_1=V_2 \approx 0$):

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \left(\frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \right) + \sum h_f + \sum h_m + h_{\text{turbine}}$$

$$h_{\text{turbine}} = (z_1 - z_2) - \sum h_f - \sum h_m = \Delta z - \frac{V_1^2}{2g} \left(f_1 \frac{L_1}{d_1} + \sum K \right) - \frac{V_2^2}{2g} \left(f_2 \frac{L_2}{d_2} \right) - \frac{V_3^2}{2g} \left(f_3 \frac{L_3}{d_3} + \sum K \right)$$

$$h_{\text{turbine}} = (150) - \frac{(9.17)^2}{2(32.2)} [0.0312(1950) + 0.5 + 2(0.39 + 0.79)] - \frac{(1.02)^2}{2(32.2)} (0.0257)(350) \\ - \frac{(4.074)^2}{2(32.2)} [(0.0283)(1000) + 8.5 + 1]$$

$$h_{\text{turbine}} = (150) - \{[82.118] + (0.145) + [9.745]\}$$

$$h_{\text{turbine}} = (150) - [92.009] = 57.99 \quad (10\text{pts})$$

Thus, the resulting turbine power is determined using the following formula:

$$P_{\text{turbine}} = \rho g Q (h_{\text{turbine}}) = (62.4)(0.2)(57.99) = 723.7 \text{ ft} \cdot \frac{\text{lb f}}{\text{s}} \div 550 \frac{\text{ft} \cdot \text{lb f/s}}{\text{hp}} = 1.316$$

The power extracted from the turbine is **1.316 hp ± 0.0263** (10pts)

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Circle the correct answer

9. (2 pt) Which of the following is NOT a common category of pumps:

- (a) Centrifugal
- (b) Impulse
- (c) Axial Flow

10. (2 pt) The *design flow rate* Q^* is the flow rate at which _____.

- (a) a pump provides its maximum discharge
- (b) a pump provides its greatest efficiency.
- (c) a pump provides the highest possible head.

11. (2 pt) This U.S. engineer invented the classical venturi device at Holyoke MA during development of the hydroelectric dam: _____.

- (a) Giovanni Venturi
- (b) Robert Manning.
- (c) Clemens Herschel

Solve and show your work.

12. Turbomachinery (30 Pts)

A pump, **geometrically similar** to the 12.5-in model in the image shown below, has a diameter of 24 in and is to develop 34 hp at BEP when pumping gasoline (not water). Determine (a) the appropriate speed, in r/min, (b) the BEP head, in ft, and (c) the BEP flow rate, in gal/min. For gasoline, $\rho = 680 \text{ kg/m}^3 = 1.32 \text{ slug/ft}^3$. Round the final answers to the nearest whole number.

From the given image, the BEP values are $H^*_1=72 \text{ ft}$; $Q^*_1= 525 \text{ gal/min}$; $\eta^*_1=0.80$

The power is determined using the following formula:

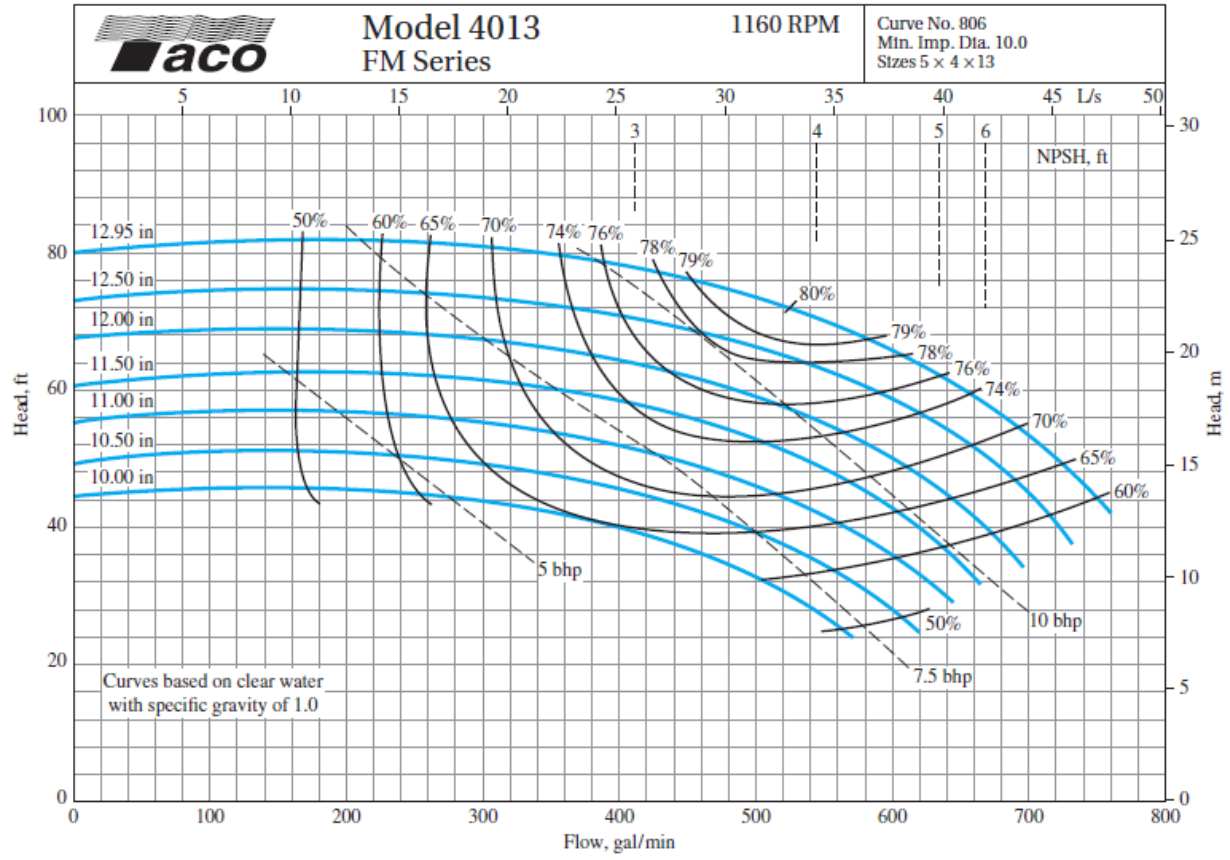
$$P_1 = \rho g Q^*_1 H^*_1 / \eta = \left(1.94 \frac{\text{slugs}}{\text{ft}^3}\right) \left(32.2 \frac{\text{ft}}{\text{s}^2}\right) \left(525 \frac{\text{gal}}{\text{min}} \div 448.8 \frac{\text{gal/min}}{\text{ft}^3/\text{s}}\right) (72 \text{ ft}) / 0.8$$
$$= 6577 \frac{\text{ft} \cdot \text{lb}_f}{\text{s}} = 11.96 \text{ hp}$$

(6pts)

Use the scaling laws to find the new speed, head, and flow rate:

$$\frac{P_2}{P_1} = \frac{34}{11.96} = \left(\frac{\rho_2}{\rho_1}\right) \left(\frac{n_2}{n_1}\right)^3 \left(\frac{D_2}{D_1}\right)^5 = \left(\frac{1.32}{1.94}\right) \left(\frac{n_2}{1160}\right)^3 \left(\frac{24}{12.95}\right)^5, \text{ solve } n_2 = 668.16 \text{ rpm} \approx 668 \text{ rpm} \quad (8\text{pts})$$
$$\frac{H_2}{H_1} = \frac{H_2}{72} = \left(\frac{n_2}{n_1}\right)^2 \left(\frac{D_2}{D_1}\right)^2 = \left(\frac{668.12}{1160}\right)^2 \left(\frac{24}{12.95}\right)^2, \text{ solve } H_2 = 82.05 \text{ ft} \approx 82 \text{ ft} \quad (8\text{pts})$$
$$\frac{Q_2}{Q_1} = \frac{Q_2}{525} = \left(\frac{n_2}{n_1}\right) \left(\frac{D_2}{D_1}\right)^3 = \left(\frac{668.12}{1160}\right) \left(\frac{24}{12.95}\right)^3, \text{ solve } Q_2 = 1924.9 \approx 1925 \text{ gal/min} \quad (8\text{pts})$$

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-----BONUS-----

Bonus (2 points): What is the difference between PDP pumps and dynamic (or momentum-change) pumps?

Dynamic pumps generally provide a higher flow rate than PDPs and a much steadier discharge but are ineffective in handling high-viscosity liquids. Dynamic pumps also generally need priming; if they are filled with gas, they cannot suck up a liquid from below into their inlet. The PDP on the other hand is self-priming for most applications. A dynamic pump can provide very high flow rates but usually with moderate pressure rises. In contrast a PDP can operate up to very high pressures but typically produces low flow rates.

Bonus (3 points): A pump from the family of Fig 11.8 (Non-dimensional plot of Performance) has $D = 28$ in and $n = 18,000$ r/min. Estimate the discharge (Q^*) for water at 60F (density =1.94 slugs/ft³) at its best efficiency.

$$n = 18,000 \text{ r/min} = 300 \text{ r/s}$$

$$Q^* = C_Q n D^3 = (0.115)(300 \text{ r/s}) \left(\frac{28 \text{ in}}{12 \text{ in}} \right)^3 = 483.3 \frac{\text{ft}^3}{\text{s}} = 196,699 \text{ gal/min}$$

Bonus (3 points): Calculate the “rigorous” specific speed (N'_s) and the “quick/lazy” specific speed (N_s) for a family of pumps from Fig 11.8.

$$N'_s = \frac{C_Q^{1/2}}{C_H^{3/4}} = \frac{(0.115)^{1/2}}{(5.0)^{3/4}} = 0.104$$

$$N_s = 17,182 N'_s = 17,182 * (0.104) = 1740$$