Homework 2 Solutions

8.1 **(*a*)** A mass balance for the dye can be written as

$$V\frac{dc_b}{dt} = E'(c_o - c_b)$$

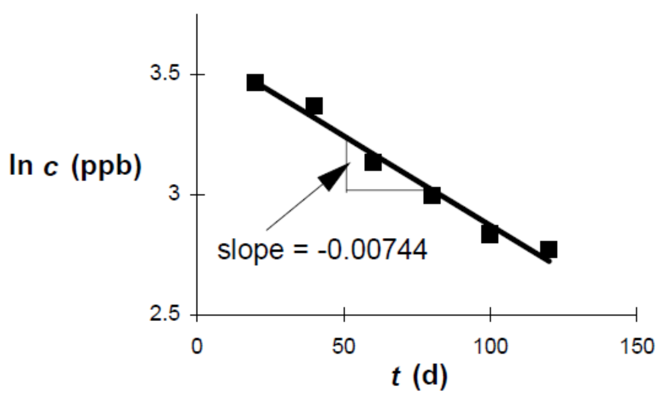
where the subscripts o and b designate the ocean and the bay, respectively. Because the ocean concentration can be assumed to be zero, the equation becomes,

$$\frac{dc_b}{dt} = -\frac{E'}{V}c_b$$

which can be solved for

$$c_b = c_{bo}e^{-(E'/V)t}$$

Therefore, a plot of $\ln(c_b)$ versus t should be linear (if our model holds) with an intercept of $\ln(c_{bo})$ and a slope of $-(E'/V)$. As shown by the following plot, the model seems valid



Therefore,

$$E' = 2.5\times10^6\ \text{m}^3(0.00744\ \text{d}^{-1}) = 18{,}599\frac{\text{m}^3}{\text{d}}$$

$$E = \frac{E'\ell}{A_c} = \frac{18599\ \text{m}^3/\text{d}(100\ \text{m})}{20\ \text{m}^2} = 92{,}955\ \frac{\text{m}^2}{\text{d}}$$

(*b*) A steady-state mass balance for the bay can be rewritten to include loading and solved for

$$W = (E'+kV)c_b = \left[18599\frac{\text{m}^3}{\text{d}} + 0.01/\text{d}(2.5\times10^6\ \text{m}^3)\right]1\frac{\text{g}}{\text{m}^3} = 0.436\times10^5\ \frac{\text{g}}{\text{d}}$$

8.5 (*a*) The bulk diffusion coefficient can be computed as

$$E' = Q\frac{s_{\text{in}} - s}{s - s_0} = 5\times10^6\,\frac{70-30}{30-7} = 8.7\times10^6\,\frac{\text{m}^3}{\text{yr}}$$

and the diffusion coefficient calculated as

$$E = \frac{E'\ell}{A_c} = \frac{8.7\times10^6(100)}{500}\,\frac{10^4\ \text{cm}^2}{\text{m}^2}\,\frac{\text{yr}}{86400\times365\ \text{s}} = 551.5\,\frac{\text{cm}^2}{\text{s}}$$

(*b*) The mass balance can be written and solved for

$$0 = W - Qc - kVc - E'c$$

$$W = Qc - kVc - E'c$$

$$W = 5\times10^6(10) + 1(981748)10 + 8695652(10) =$$

$$50\times10^7 \qquad 87\times10^6 \qquad 9.8\times10^6 \;=\; 147\times10^6\,\frac{\text{kg}}{10^3\,\text{g}} = 146{,}774\,\frac{\text{kg}}{\text{yr}}$$

(*c*)

$$\lambda = \frac{Q}{V} + \frac{E'}{V} + k = 14.95\ \text{yr}^{-1}$$

$$t_{95} = \frac{3}{14.85} = 0.2\ \text{yr}\ (= 2.4\ \text{mos})$$

9.6 (a) $\theta = Q_{10}^{0.1} = 1.7^{0.1} = 1.0545$

$$k_{27.5} = 0.2(1.0545)^{27.5-20} = 0.298$$

(*b*) Solve for the velocity

$$U = \frac{Q}{A_c} = \frac{5 \times 10^4}{400} = 125 \frac{\text{m}}{\text{d}}$$

Eq. 9.57 can be solved for

$$W = 5 \times 10^4 \frac{\text{m}^3}{\text{d}} (20 \text{ mg/m}^3) \sqrt{1 + \frac{4(0.298)1{,}000{,}000}{125^2}} \cdot \frac{\text{kg}}{10^6 \text{ mg}} = 8.79 \frac{\text{kg}}{\text{d}}$$

10.1 (*a*) After converting the english units into commensurate metric units (B = 1000 ft = 304.79 m; H = 10 ft = 3.0479 m, Q = 500 cfs = 1,223,109 m^3/d), the velocity and the loss rate can be computed as

$$U = \frac{1223109 \text{ m}^3/\text{d}}{304.79 \text{ m}(3.0479 \text{ m})} = 1316.7 \frac{\text{m}}{\text{d}} \qquad k(\text{total}) = k + \frac{v_v}{H} = 0.05 + 0.3/3.0479 = 0.148 \text{ d}^{-1}$$

which can then be used to calculate the estuary number

$$\eta = \frac{kE}{U^2} = \frac{0.148(1\times 10^6)}{1316.7^2} = 0.0856$$

Therefore, the system can be classified as being mildly advective.

(*b*) Eq. 10.24 can then be employed to compute the profile at the downstream station (x = 8 mi = 12874 m),

$$c(x,t) = \frac{10\times 10^6/928.9}{2\sqrt{\pi(1\times 10^6)t}} e^{-\frac{(12874-(1316.7)t)^2}{4(1\times 10^6)t} - 0.148t}$$

The results are shown below.