P3.1 Discuss Newton's second law (the linear momentum relation) in these three forms:

$$\sum \mathbf{F} = m\mathbf{a} \qquad \sum \mathbf{F} = \frac{d}{dt}(m\mathbf{V}) \qquad \sum \mathbf{F} = \frac{d}{dt}\left(\int_{system} \mathbf{V}\rho\, d\upsilon \right)$$

Solution: These questions are just to get the students thinking about the basic laws of mechanics. They are valid and equivalent for constant-mass systems, and we can make use of all of them in certain fluids problems, e.g. the #1 form for small elements, #2 form for rocket propulsion, but the #3 form is control-volume related and thus the most popular in this chapter.

P3.5 Water at 20°C flows through a 5-inch-diameter smooth pipe at a high Reynolds number, for which the velocity profile is given by $u \approx U_o(y/R)^{1/8}$, where U_o is the centerline velocity, R is the pipe radius, and y is the distance measured from the wall toward the centerline. If the centerline velocity is 25 ft/s, estimate the volume flow rate in gallons per minute.

Solution: The formula for average velocity in this power-law case was given in Example 3.4:

$$V_{av} = U_o \frac{2}{(1+m)(2+m)} = U_o \frac{2}{(1+1/8)(2+1/8)} = 0.837 U_o = 0.837(25) = 20.92 \frac{ft}{s}$$

$$Thus \quad Q = V_{av} A_{pipe} = [20.92 \frac{ft}{s}]\, \pi (\frac{2.5}{12} ft)^2 = 2.85 \frac{ft^3}{s} \approx \mathbf{1280} \frac{gal}{min} \qquad Ans.$$

P3.8 Three pipes steadily deliver water at 20°C to a large exit pipe in Fig. P3.8. The velocity $V_2 = 5$ m/s, and the exit flow rate $Q_4 = 120$ m³/h. Find (a) V_1; (b) V_3; and (c) V_4 if it is known that increasing Q_3 by 20% would increase Q_4 by 10%.

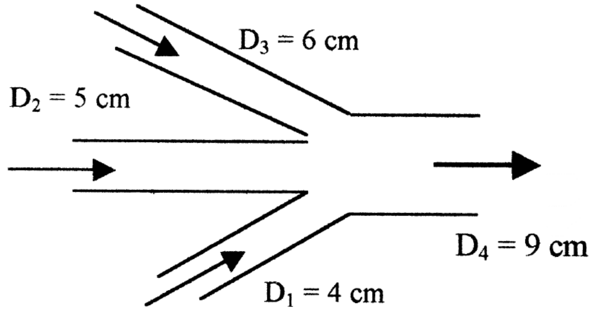


Fig. P3.8

Solution: (a) For steady flow we have $Q_1 + Q_2 + Q_3 = Q_4$, or

$$V_1A_1 + V_2A_2 + V_3A_3 = V_4A_4 \qquad (1)$$

Since $0.2Q_3 = 0.1Q_4$, and $Q_4 = (120\ \text{m}^3/\text{h})(1\ \text{h}/3600\ \text{s}) = 0.0333\ \text{m}^3/\text{s}$,

$$V_3 = \frac{Q_4}{2A_3} = \frac{(0.0333\ \text{m}^3/\text{s})}{\frac{\pi}{2}(0.06^2)} = \mathbf{5.89\ m/s} \quad \textit{Ans.}\ \text{(b)}$$

Substituting into (1),

$$V_1\left(\frac{\pi}{4}\right)(0.04^2) + (5)\left(\frac{\pi}{4}\right)(0.05^2) + (5.89)\left(\frac{\pi}{4}\right)(0.06^2) = 0.0333 \qquad \mathbf{V_1 = 5.45\ m/s} \quad \textit{Ans.}\ \text{(a)}$$

From mass conservation, $Q_4 = V_4A_4$

$$(0.0333\ \text{m}^3/\text{s}) = V_4(\pi)(0.06^2)/4 \qquad \mathbf{V_4 = 5.24\ m/s} \quad \textit{Ans.}\ \text{(c)}$$

P3.28 Air, assumed to be a perfect gas from Table A.4, flows through a long, 2-cm-diameter insulated tube. At section 1, the pressure is 1.1 MPa and the temperature is 345 K. At section 2, 67 meters further downstream, the density is 1.34 kg/m^3, the temperature 298 K, and the Mach number is 0.90. For one-dimensional flow, calculate (*a*) the mass flow; (*b*) p_2; (c) V_2; and (*d*) the change in entropy between 1 and 2. (*e*) How do you explain the entropy change?

Solution: For air, $k = 1.40$ and $R = 287$ m^2/s^2-K, hence $c_p = kR/(k-1) = 1005$ m^2/s^2-K.

(a, c) We have enough information at section 2 to calculate the velocity, hence the mass flow:

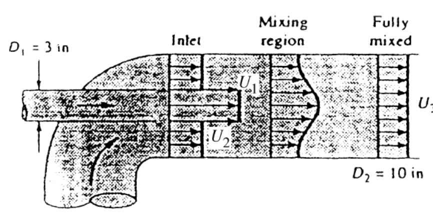
$$a_2 = \sqrt{kRT_2} = \sqrt{1.4(287)(298K)} = 346\frac{m}{s}, \text{ thus } V_2 = Ma_2\, a_2 = (0.9)(346) = \mathbf{311}\frac{m}{s} \quad Ans.(c)$$

$$\text{Then } \dot{m} = \rho_2 A_2 V_2 = (1.34\frac{kg}{m^3})[\frac{\pi}{4}(0.02\,m)^2](311\frac{m}{s}) = \mathbf{0.131}\frac{kg}{s} \quad Ans.(a)$$

(*b*) The pressure at section 2 follows from the perfect gas law:

$$p_2 = \rho_2 RT_2 = (1.34\frac{kg}{m^3})(287\frac{N-m}{kg\ K})(298\,K) = 115{,}000\frac{N}{m^2} = \mathbf{115{,}000}\ Pa \quad Ans.(b)$$

P3.36 The jet pump in Fig. P3.36 injects water at $U_1 = 40$ m/s through a 3-in pipe and entrains a secondary flow of water $U_2 = 3$ m/s in the annular region around the small pipe. The two flows become fully mixed down-stream, where U_3 is approximately constant. For steady incompressible flow, compute U_3 in m/s.



Solution: First modify the units: $D_1 = 3$ in $= 0.0762$ m, $D_2 = 10$ in $= 0.254$ m. For incompressible flow, the volume flows at inlet and exit must match:

$$Q_1 + Q_2 = Q_3, \quad \text{or: } \frac{\pi}{4}(0.0762)^2(40) + \frac{\pi}{4}[(0.254)^2 - (0.0762)^2](3) = \frac{\pi}{4}(0.254)^2 U_3$$

Solve for $\mathbf{U_3 \approx 6.33\ m/s}$ *Ans.*

P3.102 As can often be seen in a kitchen sink when the faucet is running, a high-speed channel flow (V_1, h_1) may "jump" to a low-speed, low-energy condition (V_2, h_2) as in Fig. P3.102. The pressure at sections 1 and 2 is approximately hydrostatic, and wall friction is negligible. Use the continuity and momentum relations to find h_2 and V_2 in terms of (h_1, V_1).

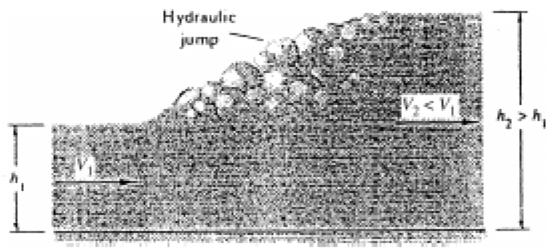


Fig. P3.102

Solution: The CV cuts through sections 1 and 2 and surrounds the jump, as shown. Wall shear is neglected. There are no obstacles. The only forces are due to hydrostatic pressure:

$$\sum F_x = 0 = \frac{1}{2}\rho g h_1(h_1 b) - \frac{1}{2}\rho g h_2(h_2 b) = \dot{m}(V_2 - V_1),$$
$$\text{where } \dot{m} = \rho V_1 h_1 b = \rho V_2 h_2 b$$

Solve for $V_2 = \mathbf{V_1 h_1/h_2}$ and $h_2/h_1 = -\frac{\mathbf{1}}{\mathbf{2}} + \frac{\mathbf{1}}{\mathbf{2}}\sqrt{\mathbf{1 + 8V_1^2/(gh_1)}}$ *Ans.*

P3.148 By neglecting friction, (*a*) use the Bernoulli equation between surfaces 1 and 2 to estimate the volume flow through the orifice, whose diameter is 3 cm. (*b*) Why is the result to part (*a*) absurd? (*c*) Suggest a way to resolve this paradox and find the true flow rate.

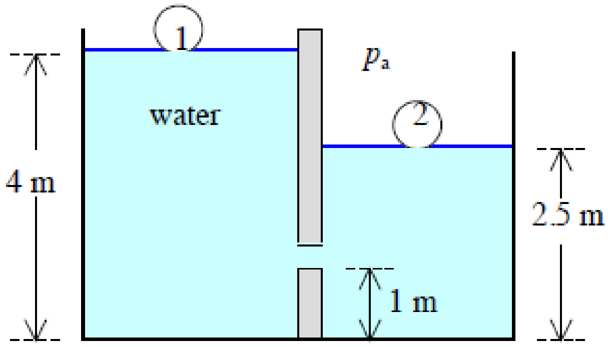


Fig. P3.148

Solution: (*a*) The incompressible Bernoulli equation between surfaces 1 and 2 yields

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_a}{9790} + \frac{0^2}{2(9..81)} + 4 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{p_a}{9790} + \frac{0^2}{2(9..81)} + 2.5$$

This gives the absurd result **4 m = 2.5 m ?** *Ans.*(*a*)

(*b*) The absurd result arises because the flow is *not frictionless*. The jet of water passing through the orifice loses all of its kinetic energy by viscous dissipation in the right-side tank. (*c*) As we shall see in Chap. 6, we add an orifice-exit *head loss* equal to the jet kinetic energy:

P3.180 Water at 20°C is pumped at 1500 gal/ min from the lower to the upper reservoir, as in Fig. P3.180. Pipe friction losses are approximated by $h_f \approx 27V^2/(2g)$, where V is the average velocity in the pipe. If the pump is 75 percent efficient, what horse-power is needed to drive it?

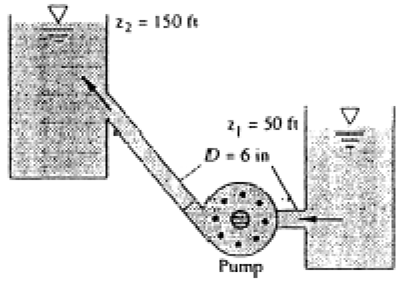


Fig. P3.180

Solution: First evaluate the average velocity in the pipe and the friction head loss:

$$Q = \frac{1500}{448.8} = 3.34\ \frac{ft^3}{s}, \quad \text{so } V = \frac{Q}{A} = \frac{3.34}{\pi(3/12)^2} = 17.0\ \frac{ft}{s} \quad \text{and} \quad h_f = 27\frac{(17.0)^2}{2(32.2)} \approx \mathbf{121\ ft}$$

Then apply the steady flow energy equation:

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f - h_p,$$

$$\text{or:} \quad 0 + 0 + 50 = 0 + 0 + 150 + 121 - h_p$$

$$\text{Thus } h_p = 221\ \text{ft}, \quad \text{so } P_{pump} = \frac{\gamma Q h_p}{\eta} = \frac{(62.4)(3.34)(221)}{0.75}$$

$$= 61600\ \frac{ft\cdot lbf}{s} \approx \mathbf{112\ hp} \quad \textit{Ans.}$$