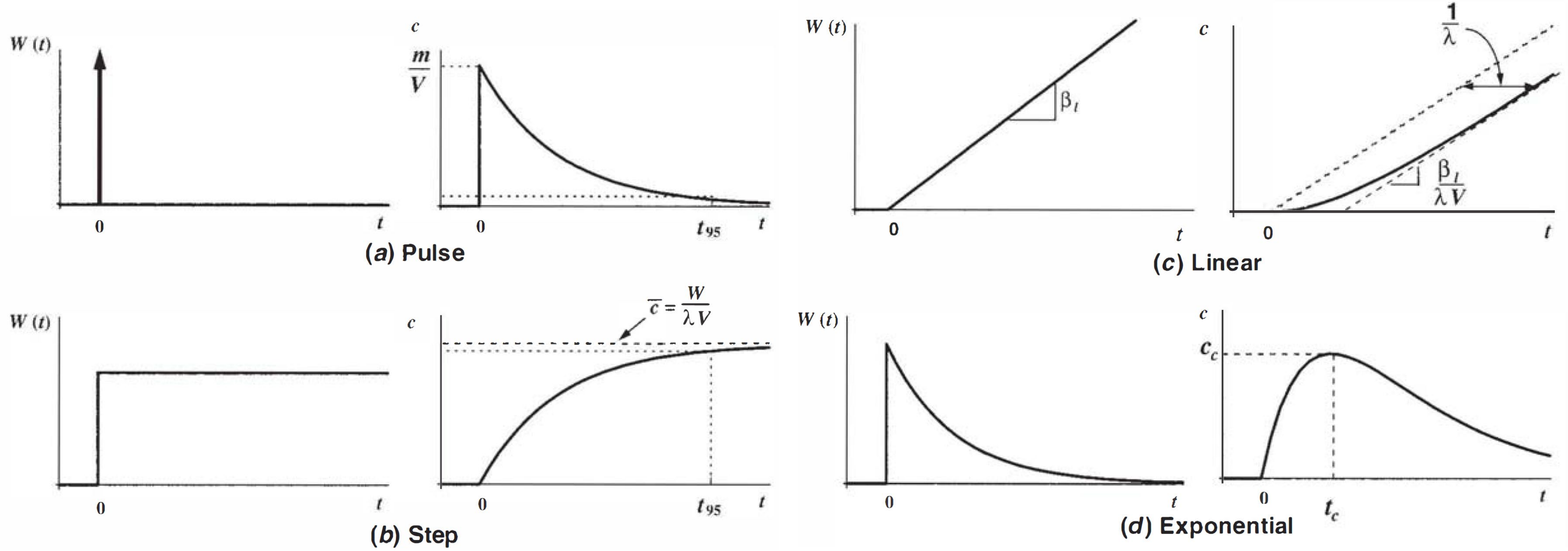
The Total Solution: Linearity and Time Shifts



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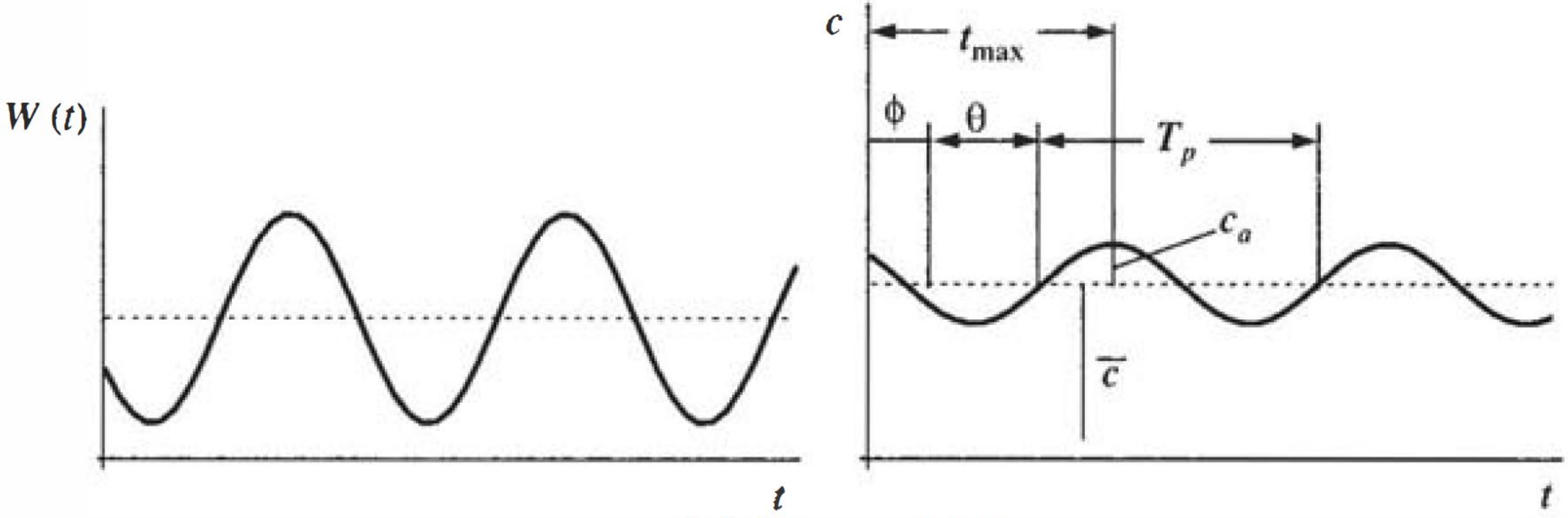

(*e*) Sinusoidal

FIGURE 4.7
A summary of the solutions developed to this point for a substance reacting with first-order kinetics. Note that the shape parameters are shown on the concentration response plots.

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These analytical solutions can be employed in tandem to assess the impact of several loading trends simultaneously. If we derive a number of particular solutions (c_{pi}), and a_i are any constants, the total solution can be derived by simple addition of the general and particular solutions.

$$c = c_g + \sum_{i=1}^{n} a_i c_{pi}$$

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If for example, there are two impulse loads, one at time zero and one at time "*a*", we could have similar functions, with one shifted "*a*" units of time (i.e c(*t*) and *c*(*t*-*a*))):

$$c = \frac{m_1}{V} e^{-\lambda t} \qquad 0 \leq t < a$$

$$c = \left(\frac{m_1}{V} e^{-\lambda t}\right) e^{-\lambda(t-a)} + \frac{m_2}{V} e^{-\lambda(t-a)} \qquad t \geq a$$

EXAMPLE 4.4. LINEAR AND EXPONENTIAL LOADINGS. O'Connor and Mueller (1970) used linear and exponential forcing functions to characterize the loadings of a conservative substance, chloride, to Lake Michigan. For example they characterized chloride loadings due to road salt by the linear model

$$W(t) = 0 \qquad t < 1930$$

$$W(t) = 13.2 \times 10^9 (t - 1930) \qquad 1930 \leq t \leq 1960$$

where $W(t)$ has units of g yr^{-1}. They used an exponential model to characterize other sources of salt that were correlated with population growth in the basin (for example municipal and industrial sources),

$$W(t) = 0 \qquad t < 1900$$

$$W(t) = 229 \times 10^9 e^{0.015(t-1900)} \qquad 1900 \leq t \leq 1960$$

Finally they considered that the lake had a background chloride concentration of 3 mg L^{-1}.

According to O'Connor and Mueller, Lake Michigan had the following average characteristics for the period from 1900 to 1960: outflow $= 49.1 \times 10^9$ $\text{m}^3\ \text{yr}^{-1}$ and volume $= 4880 \times 10^9$ m^3. Calculate the chloride concentration in Lake Michigan from 1900 through 1960.