

Homework 4 Solutions

19.3 (a) The diffusion coefficient can be estimated with a chloride balance

$$0 = Qs_{in} - Qs_2 + E'_{12}(s_1 - s_2)$$

$$E'_{12} = 4800 \frac{50 - 10}{10 - 5} = 38,400 \text{ m}^3 \text{ d}^{-1}$$

(b) The BOD in the lake and the bay can be computed, without the subdivision, as

$$c_1 = \frac{Q_{1,in}c_{1,in}[Q_2 + E'_{12} + (k + v_s / H_2)V_2] - Q_{2,in}c_{2,in}(-E'_{12} - Q_2)}{[Q_1 + E'_{12} + (k + v_s / H_1)V_1][Q_2 + E'_{12} + (k + v_s / H_2)V_2] - E'_{12}(-E'_{12} - Q_2)}$$

$$= \frac{0 - 4800(57)(38400 + 4800)}{[60477.5 + 38400 + (0.1 + 0.1/8)1.28 \times 10^7][4800 + 38400 + (0.1 + 0.1/3)1.2 \times 10^6] - 38400(-38400 - 4800)}$$

$$= 0.038 \text{ mg L}^{-1}$$

$$c_2 = \frac{Q_{2,in}c_{2,in}[Q_1 + E'_{12} + (k + v_s / H_1)V_1]_2 - Q_{1,in}c_{1,in}E'_{12}}{[Q_1 + E'_{12} + (k + v_s / H_1)V_1][Q_2 + E'_{12} + (k + v_s / H_2)V_2] - E'_{12}(E'_{12} + Q_2)}$$

$$= \frac{4800(57)[54800 + 38400 + (0.1 + 0.1/8)1.28 \times 10^7] - 0}{[60477.5 + 38400 + (0.1 + 0.1/8)1.28 \times 10^7][4800 + 38400 + (0.1 + 0.1/3)1.2 \times 10^6] - 38400(38400 + 4800)}$$

$$= 1.35 \text{ mg L}^{-1}$$

The loading and flow from the subdivision can be computed as

$$Q_{sub} = 0.568(10000) = 5678 \text{ m}^3 \text{ d}^{-1}$$

$$Q_{sub} = 113.4(10000) = 1,134,000 \text{ g d}^{-1}$$

The BOD in the lake and the bay can be computed, with the subdivision, as

$$c_1 = \frac{0 - [4800(57) + 1.134 \times 10^6](-38400 - 4800)}{[60477.5 + 38400 + (0.1 + 0.1/8)1.28 \times 10^7][4800 + 38400 + (0.1 + 0.1/3)1.2 \times 10^6] - 38400(-38400 - 4800)}$$

$$= 0.22 \text{ mg L}^{-1}$$

$$c_2 = \frac{[4800(57) + 1.134 \times 10^6][54800 + 38400 + (0.1 + 0.1/8)1.28 \times 10^7] - 0}{[60477.5 + 38400 + (0.1 + 0.1/8)1.28 \times 10^7][4800 + 38400 + (0.1 + 0.1/3)1.2 \times 10^6] - 38400(-38400 - 4800)}$$

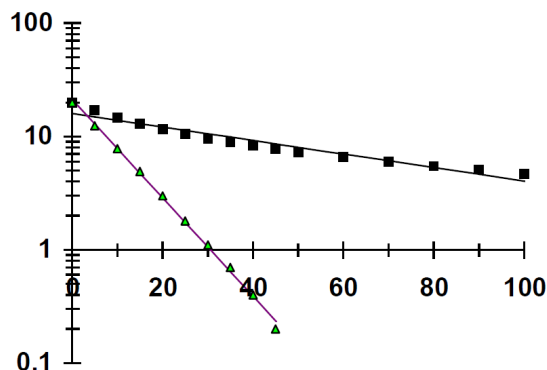
$$= 6.78 \text{ mg L}^{-1}$$

19.4 The basic approach would be to regress the ln of the particulate and dissolved components versus distance. This yields

$$\ln L_p = 3.056 - 0.1x \quad (\text{adjusted } r^2 = 0.997)$$

$$\ln L_d = 2.768 - 0.0137x \quad (\text{adjusted } r^2 = 0.937)$$

Because the fit of the particulate fraction looks O.K., the slope can be converted to a settling velocity, $v_s = 0.1 \text{ km}^{-1}(6600 \text{ m d}^{-1})(1 \text{ km}/1000 \text{ m})/2 \text{ m} = 0.33 \text{ m d}^{-1}$. [A caveat to this manipulation is that the disappearance of particulate BOD may be due to mechanisms other than settling. For example, some of it can be converted to dissolved form.]



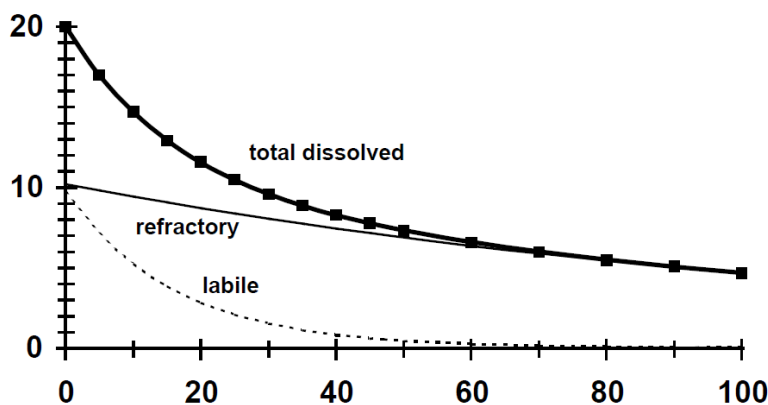
Although the r^2 's appear good for the dissolved BOD, inspection of the fit indicates that the dissolved component seems to systematically deviate from the straight line. [Aside: This is a good example of how a "good" or "high" r^2 does not always tell the whole story. It also illustrates why plots must always be developed and scrutinized]. The fits suggest that part of the dissolved component seems to decompose readily, whereas some of it seems to be refractory. This hypothesis can be expressed as an alternative model,

$$L_d = (1 - F_r)L_{do}e^{-(k_{dr}/U)x} + F_rL_{do}e^{-(k_{dl}/U)x}$$

where F_r = the fraction of the dissolved component that is refractory, and k_{dr} and k_{dl} are rates for the refractory and labile fractions, respectively. The model parameters can be estimated in a number of ways. For example, nonlinear regression can be used to estimate,

$$L_d = (1 - 0.49)L_{do}e^{-0.00784x} + 0.49L_{do}e^{-0.0618x}$$

where the fit is shown in the following plot,



Thus, from the fit, it appears that the dissolved BOD is about 50% refractory. Using the stream velocity, the exponents can be expressed as rates,

$$k_{dr} = 0.00784 / \text{km}(6600 \text{ m d}^{-1}) \frac{1 \text{ km}}{1000 \text{ m}} = 0.052 \text{ d}^{-1}$$

$$k_{dl} = 0.0618 / \text{km}(6600 \text{ m d}^{-1}) \frac{1 \text{ km}}{1000 \text{ m}} = 0.408 \text{ d}^{-1}$$

20.2 First the key model parameters can be calculated as: $V = 2.5 \times 10^6$, $Q = 3.57 \times 10^5$, $k_a = 0.11$, and $k_d = 0.158$. Next, the mass balances can be formulated in terms of population,

$$0 = \text{Pop} \times P_L - (Q + \text{Pop} \times P_Q)L - k_d VL$$

$$0 = Qo_s - (Q + \text{Pop} \times P_Q)o - k_d VL + k_a V(o_s - o)$$

where Pop = population, P_L = per capita BOD generation rate (= 125 g/cap/d), and P_Q = per capita flow generation rate (= 0.57 m³/cap/d). The first equation can be solved for

$$L = \frac{\text{Pop} \times P_L}{(Q + \text{Pop} \times P_Q) + k_d V}$$

This result can be substituted into the second equation which can be solved (with $c = 6$ mg/L) for Pop = 14,417 people. This can in turn be used to compute

$$W = \text{Pop} \times P_L = 14417 \times 125 = 1.8 \times 10^6 \frac{\text{g}}{\text{d}}$$

Note that the corresponding waste flow is

$$Q_w = \text{Pop} \times P_Q = 14417 \times 0.57 = 8217 \frac{\text{m}^3}{\text{d}}$$

Finally, the inflow concentration can be calculated as

$$c_{\text{in}} = \frac{1.8 \times 10^6}{3.57 \times 10^5 + 8217} = 4.93 \frac{\text{mg}}{\text{L}}$$

20.9 (a) According to Fig. 20.7, the Owens-Gibbs formula should hold,

$$k_a = 5.32 \frac{0.4^{0.67}}{0.3^{1.85}} 1.024^{23-20} = 28.68 \text{ d}^{-1}$$

(b) For carbon dioxide, Eq. 20.58 provides a first estimate

$$k_{a,CO_2} = 28.68 \left(\frac{32}{44} \right)^{0.25} = 26.5 \text{ d}^{-1}$$

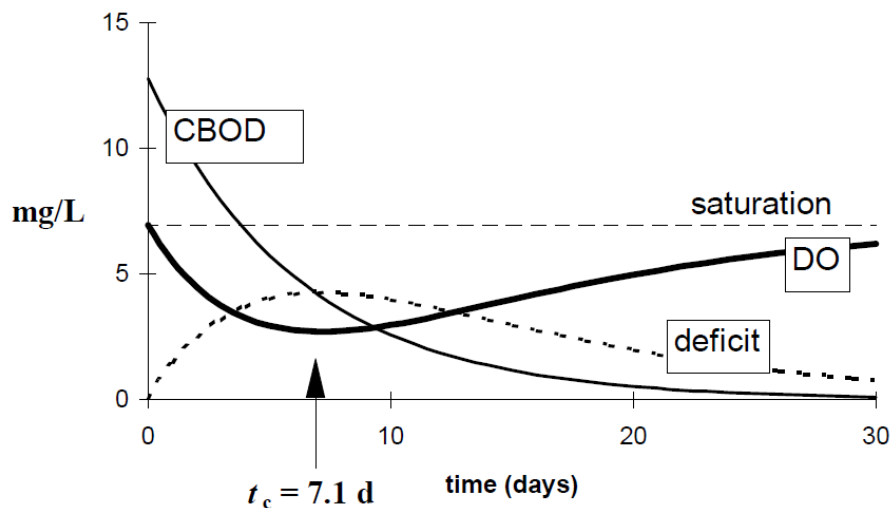
21.1 The amount of CBOD spilled can be calculated as

$$M_{CBOD} = 30000 \text{ L} \left(100 \frac{\text{g-glucose}}{\text{L}} \right) \frac{6 \text{ mole} \cdot 32 \text{ gO}_2 / \text{mole}}{1 \text{ mole} \cdot (6 \cdot 12 + 12 \cdot 1 + 6 \cdot 16) \text{ g-glucose/mole}} = 3.2 \times 10^6 \text{ g-CBOD}$$

Mass balances for CBOD and oxygen can be written and solved for

$$L = L_o e^{-(1/\tau_w + k_d)t} \quad o = o_s - \frac{k_d L_o}{k_d - k_a} \left(e^{-(1/\tau_w + k_a)t} - e^{-(1/\tau_w + k_d)t} \right)$$

The remaining model parameters can be evaluated as: $o_s = 6.942$, $k_a = 0.08925$, $k_d = 0.126$. These can be substituted to give the following profiles:



22.1 The solution for a distributed source with no flow is

$$c = \frac{S_d''}{A_c k} \left(1 - e^{-\frac{k}{U}x} \right)$$

where S_d'' = the distributed source expressed as a line load (recall Fig. 22.2a) with units of $\text{mg m}^{-1} \text{d}^{-1}$. This equation can then be solved for

$$S_d'' = \frac{c A_c k}{1 - e^{-\frac{k}{U}x}} = \frac{2(10)(0.693/5)}{\frac{0.693/5}{0.1(86400)} - 1} = 18.7 \frac{\text{mg}}{\text{m d}}$$

22.5 A good way to solve this problem involves the control-volume approach implemented on a spreadsheet. Flow balances for each segment can be written as

$$Q_i = Q_{i-1} + Q_{di} + Q_{pi}$$

where Q_{di} = the natural distributed inflow for segment i ($=q\Delta x$), Δx = volume length, and Q_{pi} = the new distributed flow due to the squatters, which can be calculated as

$$Q_{pi} = \frac{\text{Pop} \times P_q \times \Delta x}{x_{\text{tot}}}$$

where Pop = the squatter population, P_q = the per-capita rate of flow generation for a developing country (recall Table 19.2), and x_{tot} = the total distance of the stretch (i.e., 20000 m).

The depth can then be calculated based on Manning's formula. This can be done on a spreadsheet by setting up the following circular calculation (i.e., one-point iteration or successive substitution as described in Chapra and Canale 1988),

$$H = \left[\frac{Qn}{S^{1/2}} \frac{(B + 2H)^{2/3}}{B^{5/3}} \right]^{3/5}$$

Once the depth is determined, the cross-sectional area ($A_c = BH$), velocity ($U = Q/A_c$), reaeration (O'Connor-Dobbins) and decay rates (Eq. 19.28) can be determined for each cell.

Next, BOD and oxygen balances can be written for each volume,

$$0 = Q_{i-1}L_{i-1} + Q_{di}L_d + Q_{pi}L_p - k_{di}V_iL_i - Q_iL_i$$

$$0 = Q_{i-1}o_{i-1} + Q_{di}o_d + Q_{pi}o_p + k_{ai}V_i(o_s - o_i) - k_{di}V_iL_i - Q_i o_i$$

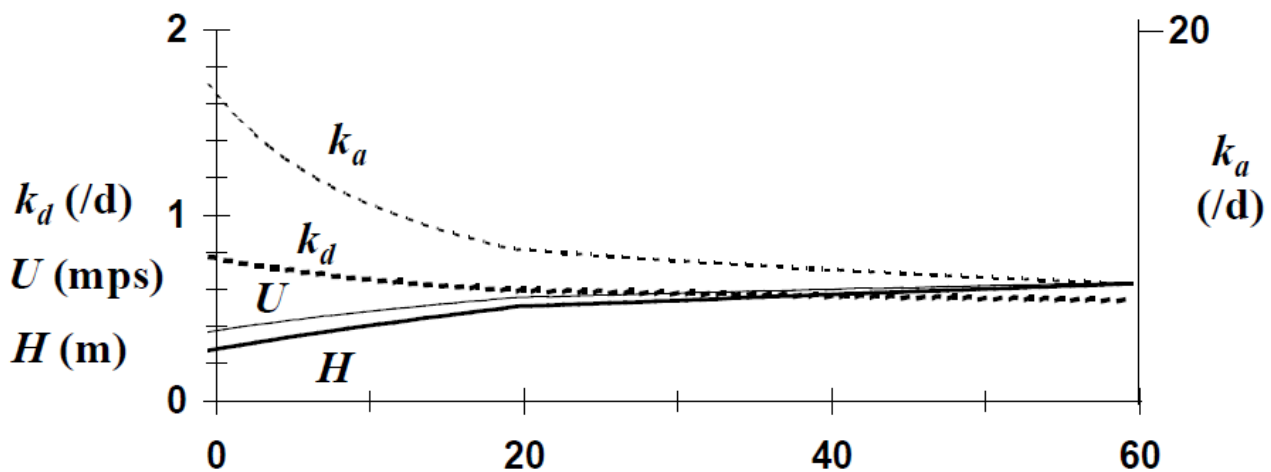
which can be solved serially for

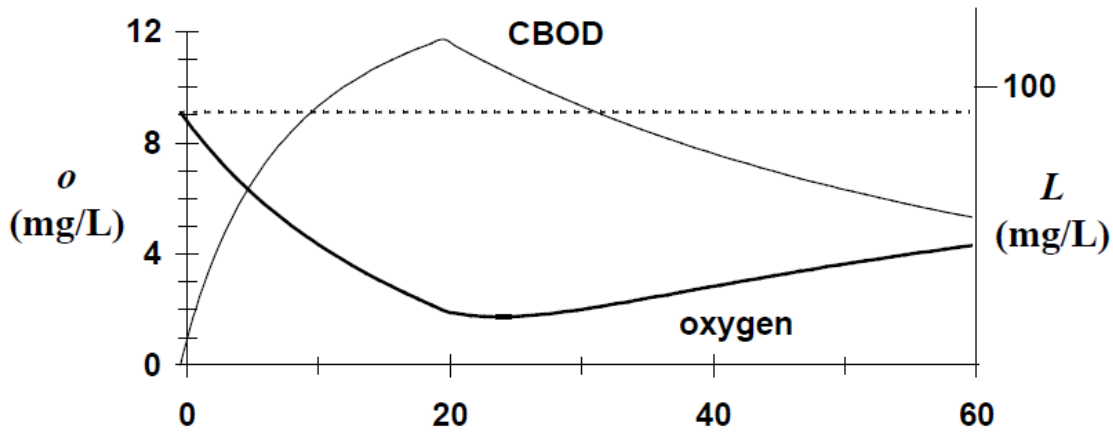
$$L_i = \frac{Q_{i-1}L_{i-1} + Q_{di}L_d + Q_{pi}L_p}{Q_i + k_{di}V_i}$$

$$o_i = \frac{Q_{i-1}o_{i-1} + Q_{di}o_d + Q_{pi}o_p + k_{ai}V_i o_s - k_{di}V_i L_i}{Q_i + k_{ai}V_i}$$

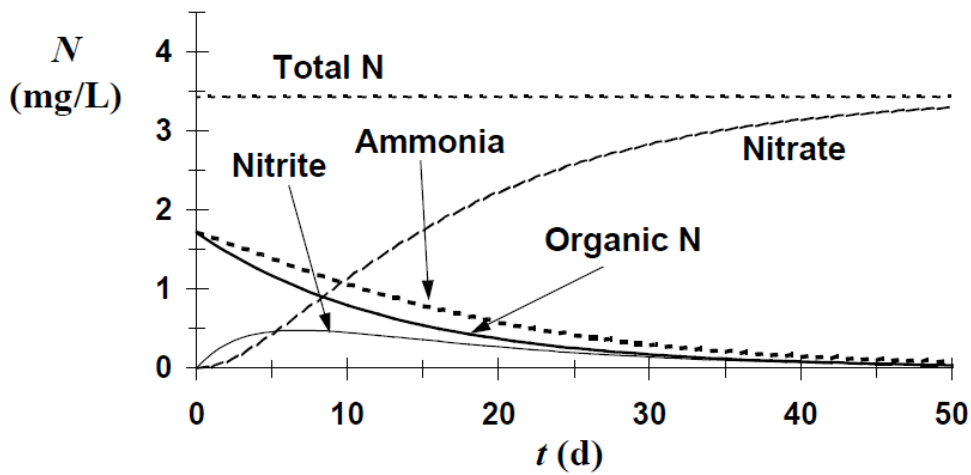
Note that the concentration of BOD from the squatters can be computed from Table 19.2 as $60/0.19 = 315.8$ mg/L.

After this calculation is implemented on a spreadsheet, a squatter population can be determined that leads to a minimum oxygen of 2 mg/L. For the present case, I determined a population of 1.1×10^6 people. Some of the resulting parameters and the variable distributions are shown below:





23.1 Mass balances at the injection point can be used to generate initial conditions for organic N and ammonium as $(2/7)6 = 1.714$ mg/L. Then the mass balance equations (Eq. 23.9 through 23.12) can be integrated (either numerically or with Eqs. 23.14 through 23.17) to give the profiles in the following plot,



The hydrogeometric parameters can be used in conjunction with Manning's equation to determine that $U = 0.664$ mps. This can be used to express the profiles on a distance rather than time reference. Thus, the 50 day series shown above would correspond to a distance of,

$$x = Ut = 0.664 \frac{\text{m}}{\text{s}} (50 \text{ d}) 86400 \frac{\text{s}}{\text{d}} \frac{\text{km}}{1000 \text{ m}} = 2870 \text{ km}$$

23.3 Equations 23.15 and 23.16 can be substituted into Eq. 23.13 to give,

$$\begin{aligned} \frac{dD}{dt} + k_a D = r_{oa} k_{ai} \left[N_{ao} e^{-k_{ai}t} + \frac{k_{oa} N_{oo}}{k_{ai} - k_{oa}} \left(e^{-k_{oa}t} - e^{-k_{ai}t} \right) \right] \\ + r_{oi} k_{in} \left[\frac{k_{ai} N_{ao}}{k_{in} - k_{ai}} \left(e^{-k_{ai}t} - e^{-k_{in}t} \right) + \frac{k_{ai} k_{oa} N_{oo}}{(k_{ai} - k_{oa})} \left[\frac{e^{-k_{oa}t} - e^{-k_{in}t}}{(k_{in} - k_{oa})} - \frac{e^{-k_{ai}t} - e^{-k_{in}t}}{(k_{in} - k_{ai})} \right] \right] \end{aligned}$$

The right-hand-side can be made manageable by clustering terms to give

$$\frac{dD}{dt} + k_a D = C_{ai} e^{-k_{ai}t} + C_{oa} e^{-k_{oa}t} + C_{in} e^{-k_{in}t}$$

$$C_{ai} = r_{oa} k_{ai} N_{ao} - r_{oa} k_{ai} \frac{k_{oa} N_{oo}}{k_{ai} - k_{oa}} + r_{oi} k_{in} \frac{k_{ai} N_{ao}}{k_{in} - k_{ai}} - r_{oi} k_{in} \frac{k_{ai} k_{oa} N_{oo}}{(k_{ai} - k_{oa})(k_{in} - k_{ai})}$$

$$C_{oa} = r_{oa} k_{ai} \frac{k_{oa} N_{oo}}{k_{ai} - k_{oa}} + r_{oi} k_{in} \frac{k_{ai} k_{oa} N_{oo}}{(k_{ai} - k_{oa})(k_{in} - k_{oa})}$$

$$C_{in} = r_{oi} k_{in} \frac{k_{ai} N_{ao}}{k_{ai} - k_{in}} - r_{oi} k_{in} \frac{k_{ai} k_{oa} N_{oo}}{(k_{ai} - k_{oa})(k_{in} - k_{oa})} + r_{oi} k_{in} \frac{k_{ai} k_{oa} N_{oo}}{(k_{ai} - k_{oa})(k_{in} - k_{ai})}$$

Then the solution simply becomes (recall Eq. 4.18),

$$D = D_0 e^{-k_a t} + \frac{C_{ai}}{k_a - k_{ai}} \left(e^{-k_{ai}t} - e^{-k_a t} \right) + \frac{C_{oa}}{k_a - k_{oa}} \left(e^{-k_{oa}t} - e^{-k_a t} \right) + \frac{C_{in}}{k_a - k_{in}} \left(e^{-k_{in}t} - e^{-k_a t} \right)$$

24.1 First, the rate can be converted to a volumetric basis and oxygen units as in

$$P_{m,o} = r_{oc} \frac{P'_{m,c}}{H} = 2.67 \frac{10}{0.8} = 33.375 \frac{\text{gO}}{\text{m}^3 \text{d}}$$

Then the average daily and daylight photosynthesis rates can be determined as,

$$P = 33.375 \left(\frac{2(0.58333)}{\pi} \right) = 12.394 \frac{\text{gO}}{\text{m}^3 \text{d}}$$

$$P_a = 33.375 \left(\frac{2}{\pi} \right) = 21.25 \frac{\text{gO}}{\text{m}^3 \text{d}}$$

24.5 Solar noon is $7 + 10/2 = 12$. Using Fig. 24.5, the lag can be determined as $\phi = 3.85$ hrs. This means that the peak occurs at about 1550 or 3:50 P.M.