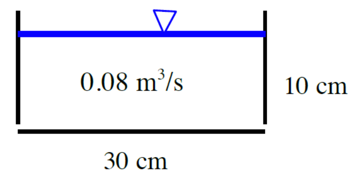
P10.2 Water at 20°C flows in a 30-cm-wide rectangular channel at a depth of 10 cm and a flow rate of 95,500 cm^3/s. Estimate (*a*) the Froude number; and (*b*) the Reynolds number.

Solution: For water, take $\rho = 998\ kg/m^3$ and $\mu = 0.001$ kg/m-s. The surface wave speed is



$$c_o = \sqrt{gy} = \sqrt{(9.81m/s^2)(0.1m)} = 0.99\,m/s$$

The average velocity is determined from the given flow rate and area:

$$V = \frac{Q}{A} = \frac{(95{,}500\ cm^3/s)}{(30cm)(10cm)} = 318.33\ \frac{cm}{s} = 3.18\ \frac{m}{s}$$

$$\text{Froude number:} \quad Fr = \frac{V}{c_o} = \frac{3.18\ m/s}{0.99\,m/s} = \mathbf{3.22} \quad \text{(supercritical)} \quad \textit{Ans.(a)}$$

The Reynolds number should be, for this writer, based upon hydraulic radius:

$$R_h = \frac{A}{P} = \frac{(0.3m)(0.1m)}{(0.3+0.1+0.1m)} = 0.06\,m$$

$$\text{Reynolds number:} \quad Re_{R_h} = \frac{\rho V R_h}{\mu} = \frac{(998)(2.67)(0.06)}{(0.001)} = 190{,}618 \ (\textit{turbulent}) \quad \textit{Ans.(b)}$$

P10.16 Water flows in a brickwork rectangular channel 2.6m wide, on a slope of 5 m per km. (*a*) Find the flow rate when the normal depth is 50 cm. (*b*) If the normal depth remains 50 cm, find the channel width which will triple the flow rate. Comment on this result.

Solution: The rectangular channel is fairly easy to set up for uniform flow. For brickwork, n = 0.015. (a) Work out the hydraulic radius and then the uniform flow rate:

$$R_h = \frac{by}{b+2y} = \frac{(2.6)(0.5)}{2(0.5)+(2.6)} = 0.361\,m, \quad Q = \frac{1}{0.015}[2.6(0.5)](0.361)^{2/3}\left(\frac{5}{1000}\right)^{1/2} = 3.11\ \frac{m^3}{s} \quad \textit{Ans.(a)}$$

(*b*) Triple the flow rate to 3(3.11) =9.33 m^3/s. Then the new Chézy formula is:

$$Q = 9.33 = \frac{1}{0.015}[b(0.5)]\left[\frac{b(0.5)}{2(0.5)+b}\right]^{2/3}\left(\frac{5}{1000}\right)^{1/2}; \quad \text{Solve} \quad b = \mathbf{6.87\ m} \quad \textit{Ans.(b)}$$

Uniform flow is nonlinear with width: Multiplying b by 2.6 triples the flow rate.

P10.31 An unfinished-concrete 6-ft-diameter sewer pipe flows half full. What is the appropriate slope to deliver 70,000 gal/min of water in uniform flow?

Solution: For unfinished concrete, from Table 10.1, $n \approx 0.022$. For a half-full circle,

$$A = \frac{\pi}{2}R^2 = \frac{\pi}{2}(3\,ft)^2 = 14.14\,ft^2;\ P = \pi R = \pi(3\,ft) = 9.43\,ft;\ R_h = \frac{A}{P} = 1.50\,ft$$

$$Q = 70{,}000\ \frac{gal}{min} = 155.9\ \frac{ft^3}{s} = \frac{1.486}{n} A R_h^{2/3}\sqrt{S_o} = (\frac{1.486}{0.022})(14.14\,ft^2)(1.50\,ft)^{2/3}\sqrt{S_o}$$

Solve for slope $S_o \approx$ **0.015529** *Ans.*

P10.51 An unfinished concrete duct, of diameter 3.7 m, is flowing half-full at 60 m^3/s. (*a*) Is this a critical flow? If not, what is (*b*) the critical flow rate, (*c*) the critical slope, and (*d*) the Froude number? (*e*) If the flow is uniform, what is the slope of the duct?

Solution: For unfinished concrete, from Table 10.1, $n = 0.014$. (*b*) Compute the critical flow rate:

$$Q_c = [\frac{gA_c^3}{b_o}]^{1/2} = [\frac{(9.81m/s^2)(\pi\{1.85m\}^2)^3}{3.7m}]^{1/2} = 57.4\ \frac{m^3}{s} \quad Ans.(b)$$

This is less than the given $Q = 60$ m^3/s, so this is a *supercritical* flow. *Ans.*(*a*)

(*d*) The average velocity is $Q/A = (60)/[\pi(1.85)^2] = 5.58$ m/s, but the critical velocity is less:

$$V_c = \sqrt{\frac{gA_c}{b_o}} = \sqrt{\frac{(9.81)[\pi(1.85)^2]}{3.7}} = 5.34\ \frac{m}{s}; \quad \text{thus } Fr = \frac{5.58}{5.34} = 1.045\ Ans.(d)$$

(*c*) The critical slope is computed from Eq. (10.38):

$$S_c = \frac{n^2 V_c^2}{\alpha^2 R_h^{4/3}} = \frac{(0.014)^2(5.34)^2}{(1.0)^2(3.7/4)^{4/3}} = 0.0062 \quad Ans.(c)$$

(*e*) If this duct is in uniform flow at the higher rate, its actual slope is much greater:

$$Q = 60\ \frac{m^3}{s} = \frac{1}{n}AR_h^{2/3}\sqrt{S_o} = \frac{1}{n}[\pi(1.85)^2](0.925)^{2/3}\sqrt{S_o};\ solve\ S_o \approx 0.0068 \quad Ans.(e)$$

P10.73 In Fig. P10.69, let y_1 = 6 ft and the gate width b = 8 ft. Find (a) the gate opening H that would allow a free-discharge flow of 30,000 gal/min; and (b) the depth y_2.

Solution: (a) The gate-opening problem is handled by Eq. (10.41):

$$Q = C_d H b \sqrt{2 g y_1} \ , \quad where \quad C_d \approx \frac{0.61}{\sqrt{1 + 0.61 H / y_1}}$$

Everything is known except H, and we know $C_d \approx 0.6$, so very little iteration is necessary:

$$Q = 30{,}000 \frac{gal}{min} = 66.84 \frac{ft^3}{3} = [\frac{0.61}{\sqrt{1+0.61 H / 6\,ft}}] H (8\,ft) \sqrt{2(32.2)(6\,ft)}$$

Solve for $H \approx$ **0.72** ft *Ans.*(a)