P1.4 Sand, and other granular materials, definitely *flow*, that is, you can pour them from a container or a hopper. There are whole textbooks on the "transport" of granular materials [54]. Therefore, is sand a *fluid*? Explain.

Solution: Granular materials do indeed *flow*, at a rate that can be measured by "flowmeters". But they are *not* true fluids, because they can support a small shear stress without flowing. They may rest at a finite angle without flowing, which is not possible for liquids (see Prob. P1.3). The maximum such angle, above which sand begins to flow, is called the *angle of repose*. A familiar example is sugar, which pours easily but forms a significant angle of repose on a heaping spoonful. The physics of granular materials are complicated by effects such as particle cohesion, clumping, vibration, and size segregation. See Ref. 54 to learn more.

P1.16 Test the dimensional homogeneity of the boundary-layer *x*-momentum equation:

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\rho g_x+\frac{\partial \tau}{\partial y}$$

Solution: This equation, like **all** theoretical partial differential equations in mechanics, is dimensionally homogeneous. Test each term in sequence:

$$\left\{\rho u\frac{\partial u}{\partial x}\right\}=\left\{\rho v\frac{\partial u}{\partial y}\right\}=\frac{M}{L^3}\frac{L}{T}\frac{L/T}{L}=\left\{\mathbf{\frac{M}{L^2T^2}}\right\};\quad \left\{\frac{\partial p}{\partial x}\right\}=\frac{M/LT^2}{L}=\left\{\mathbf{\frac{M}{L^2T^2}}\right\}$$

$$\{\rho g_x\}=\frac{M}{L^3}\frac{L}{T^2}=\left\{\mathbf{\frac{M}{L^2T^2}}\right\};\quad \left\{\frac{\partial \tau}{\partial x}\right\}=\frac{M/LT^2}{L}=\left\{\mathbf{\frac{M}{L^2T^2}}\right\}$$

All terms have dimension $\{ML^{-2}T^{-2}\}$. This equation may use *any* consistent units.

P1.20 Books on porous media and atomization claim that the viscosity μ and surface tension Υ of a fluid can be combined with a characteristic velocity U to form an important dimensionless parameter. (*a*) Verify that this is so. (*b*) Evaluate this parameter for water at 20°C and a velocity of 3.5 cm/s. NOTE: Extra credit if you know the name of this parameter.

Solution: We know from Table 1.2 that $\{\mu\} = \{ML^{-1}T^{-1}\}$, $\{U\} = \{LT^{-1}\}$, and $\{\Upsilon\} = \{FL^{-1}\} = \{MT^{-2}\}$. To eliminate mass {M}, we must divide μ by Υ, giving $\{\mu/\Upsilon\} = \{TL^{-1}\}$. Multiplying by the velocity will thus cancel all dimensions:

$$\frac{\mu U}{\Upsilon} \text{ is dimensionless, as is its inverse, } \frac{\Upsilon}{\mu U} \qquad \textit{Ans.(a)}$$

The grouping is called the *Capillary Number*. (*b*) For water at 20°C and a velocity of 3.5 cm/s, use Table A.3 to find $\mu = 0.001$ kg/m-s and $\Upsilon = 0.0728$ N/m. Evaluate

$$\frac{\mu U}{\Upsilon} = \frac{(0.001 kg/m-s)(0.035 m/s)}{(0.0728 kg/s^2)} = \mathbf{0.00048}\,,\ \frac{\Upsilon}{\mu U} = \mathbf{2080} \qquad \textit{Ans.(b)}$$

P1.38 In Fig. P1.38, if the fluid is glycerin at 20°C and the width between plates is 6 mm, what shear stress (in Pa) is required to move the upper plate at V = 5.5 m/s? What is the flow Reynolds number if "L" is taken to be the distance between plates?

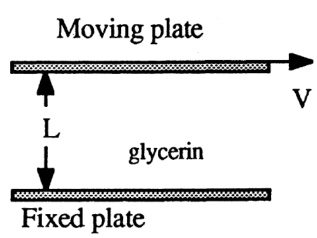

Fig. P1.38

Solution: (a) For glycerin at 20°C, from Table 1.4, $\mu \approx 1.5\ N\cdot s/m^2$. The shear stress is found from Eq. (1) of Ex. 1.8:

$$\tau = \frac{\mu V}{h} = \frac{(1.5\ Pa\cdot s)(5.5\ m/s)}{(0.006\ m)} \approx \mathbf{1380\ Pa} \qquad \textit{Ans.}\ (a)$$

The density of glycerin at 20°C is 1264 kg/m^3. Then the Reynolds number is defined by Eq. (1.24), with L = h, and is found to be decidedly laminar, Re < 1500:

$$Re_L = \frac{\rho VL}{\mu} = \frac{(1264\ kg/m^3)(5.5\ m/s)(0.006\ m)}{1.5\ kg/m\cdot s} \approx \mathbf{28} \qquad \textit{Ans.}\ (b)$$

Fig. P1.54

***P1.54** A disk of radius R rotates at angular velocity Ω inside an oil container of viscosity μ, as in Fig. P1.54. Assuming a linear velocity profile and neglecting shear on the outer disk edges, derive an expression for the viscous torque on the disk.

Solution: At any $r \le R$, the viscous shear $\tau \approx \mu\Omega r/h$ on both sides of the disk. Thus,

$$d(\text{torque}) = dM = 2r\,\tau\,dA_w = 2r\frac{\mu\Omega r}{h}2\pi r\,dr,$$

$$\text{or:}\quad M = 4\pi\frac{\mu\Omega}{h}\int_0^R r^3\,dr = \mathbf{\frac{\pi\mu\Omega R^4}{h}}\quad \textit{Ans.}$$

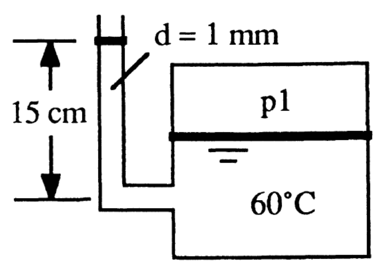

Fig. P1.65

1.65 The system in Fig. P1.65 is used to estimate the pressure p_1 in the tank by measuring the 15-cm height of liquid in the 1-mm-diameter tube. The fluid is at 60°C. Calculate the true fluid height in the tube and the percent error due to capillarity if the fluid is (a) water; and (b) mercury.

Solution: This is a somewhat more realistic variation of Ex. 1.9. Use values from that example for contact angle θ:

(a) Water at 60°C: $\gamma \approx 9640\ \text{N/m}^3$, $\theta \approx 0°$:

$$h = \frac{4Y\cos\theta}{\gamma D} = \frac{4(0.0662\ \text{N/m})\cos(0°)}{(9640\ \text{N/m}^3)(0.001\ \text{m})} = 0.0275\ \text{m},$$

or: $\Delta h_{true} = 15.0 - 2.75\ \text{cm} \approx$ **12.25 cm (+22% error)** *Ans.* (a)

(b) Mercury at 60°C: $\gamma \approx 132200\ \text{N/m}^3$, $\theta \approx 130°$:

$$h = \frac{4Y\cos\theta}{\gamma D} = \frac{4(0.47\ \text{N/m})\cos 130°}{(132200\ \text{N/m}^3)(0.001\ \text{m})} = -0.0091\ \text{m},$$

or: $\Delta h_{true} = 15.0 + 0.91 \approx$ **15.91 cm (−6% error)** *Ans.* (b)

P1.73 A small submersible moves at velocity V in 20°C water at 2-m depth, where ambient pressure is 131 kPa. Its critical cavitation number is Ca ≈ 0.25. At what velocity will cavitation bubbles form? Will the body cavitate if V = 30 m/s and the water is cold (5°C)?

Solution: From Table A-5 at 20°C read p_v = 2.337 kPa. By definition,

$$Ca_{crit} = 0.25 = \frac{2(p_a - p_v)}{\rho V^2} = \frac{2(131000 - 2337)}{(998\ kg/m^3)V^2}, \quad \text{solve } V_{crit} \approx \mathbf{32.1\ m/s} \quad \textit{Ans.}\ (a)$$

If we decrease water temperature to 5°C, the vapor pressure reduces to 863 Pa, and the density changes slightly, to 1000 kg/m^3. For this condition, if V = 30 m/s, we compute:

$$Ca = \frac{2(131000 - 863)}{(1000)(30)^2} \approx 0.289$$

This is *greater* than 0.25, therefore the body **will not cavitate for these conditions**. *Ans.* (b)